

Introduction

MPRI 2–6: Abstract Interpretation,
application to verification and static analysis

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Course 01b

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Motivating program verification

The cost of software failure

- **Patriot MIM-104** failure, 25 February 1991
(death of 28 soldiers¹)
- **Ariane 5** failure, 4 June 1996
(cost estimated at more than 370 000 000 US\$²)
- **Toyota** electronic throttle control system failure, 2005
(at least 89 death³)
- **Heartbleed** bug in OpenSSL, April 2014
- ...
- economic cost of software bugs is tremendous⁴

¹R. Skeel. "Roundoff Error and the Patriot Missile". SIAM News, volume 25, nr 4.

²M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.

³CBSNews. Toyota "Unintended Acceleration" Has Killed 89. 20 March 2014.

⁴NIST. Software errors cost U.S. economy \$59.5 billion annually. Tech. report, NIST Planning Report, 2002.

Zoom on: Ariane 5, Flight 501



Maiden flight of the Ariane 5 Launcher, 4 June 1996.

Zoom on: Ariane 5, Flight 501



40s after launch. . .

Zoom on: Ariane 5, Flight 501

Cause: software error⁵

- arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types⁶

```
P_M_DERIVE(T_ALG.E_BH) :=  
  UC_16S_EN_16NS (TDB.T_ENTIER_16S  
    ((1.0/C_M_LSB_BH) * G_M_INFO_DERIVE(T_ALG.E_BH)));
```

- software exception not caught
⇒ computer switched off
- all backup computers run the same software
⇒ all computers switched off, no guidance
⇒ rocket self-destructs

⁵J.-L. Lions et al., Ariane 501 Inquiry Board report.

⁶J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d'informatique de l'ENS, 2010.

How can we avoid such failures?

- Choose a safe programming language.
C (low level) / Ada, Java (high level)
- Carefully design the software.
many software development methods exist
- Test the software extensively.

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yet, Ariane 5 software is written in Ada

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yet, critical embedded software follow strict development processes

- Test the software extensively.

yet, the erroneous code was well tested... on Ariane 4

⇒ **not sufficient!**

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yet, the erroneous code was well tested... on Ariane 4

⇒ **not sufficient!**

We should use **formal methods**.

provide rigorous, mathematical insurance

Proving program properties

Invariants and programs

```
assume X in [0,1000];
```

```
I := 0;
```

```
while I < X do
```

```
    I := I + 2;
```

```
assert I in [0,?]
```

Goal: find a bound property, sufficient to express the absence of overflow

⁷ R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

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Invariants and programs

```
assume X in [0,1000];  
{X ∈ [0,1000]}  
I := 0;  
{X ∈ [0,1000], I = 0}  
while I < X do  
    {X ∈ [0,1000], I ∈ [0,998]}  
    I := I + 2;  
    {X ∈ [0,1000], I ∈ [2,1000]}  
{X ∈ [0,1000], I ∈ [0,1000]}  
assert I in [0,1000]
```



Robert Floyd⁷

invariant: property true of all the executions of the program

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Invariants and programs

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while I < X do  
  {X ∈ [0,1000], I ∈ [0,998]}  
  I := I + 2;  
  {X ∈ [0,1000], I ∈ [2,1000]}  
{X ∈ [0,1000], I ∈ [0,1000]}  
assert I in [0,1000]
```



Robert Floyd⁷

invariant: property true of all the executions of the program

issue: if $I = 997$ at a loop iteration, $I = 999$ at the next

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Invariants and programs

```
assume X in [0,1000];  
{X ∈ [0,1000]}  
I := 0;  
{X ∈ [0,1000], I = 0}  
while I < X do  
  {X ∈ [0,1000], I ∈ {0, 2, ..., 996, 998}}  
  I := I + 2;  
  {X ∈ [0,1000], I ∈ {2, 4, ..., 998, 1000}}  
{X ∈ [0,1000], I ∈ {0, 2, ..., 998, 1000}}  
assert I in [0,1000]
```



Robert Floyd⁷

inductive invariant: invariant that can be proved to hold by induction on loop iterates

(if $I \in S$ at a loop iteration, then $I \in S$ at the next loop iteration)

⁷R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

$$\frac{}{\{P[e/X]\} X := e \{P\}} \quad \frac{\{P\} C_1 \{R\} \quad \{R\} C_2 \{Q\}}{\{P\} C_1; C_2 \{Q\}}$$

$$\frac{\{P \& b\} C \{P\}}{\{P\} \text{while } b \text{ do } C \{P \& \neg b\}}$$

...



Tony Hoare⁸

- sound logic to prove program properties, (rel.) complete
- proofs can be partially automated (at least proof checking)
(e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)

⁸C. A. R. Hoare. "An Axiomatic Basis for Computer Programming". *Commun. ACM* 12(10): 576–580 (1969).

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- sound logic to prove program properties, (rel.) complete
- proofs can be partially automated (at least proof checking)
(e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)
- requires annotations and interaction with a prover
even **manual annotation is not practical for large programs**

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A calculus of program properties

$$wlp(X := e, P) \stackrel{\text{def}}{=} P[e/X]$$

$$wlp(C_1; C_2, P) \stackrel{\text{def}}{=} wlp(C_1, wlp(C_2, P))$$

$$wlp(\text{while } e \text{ do } C, P) \stackrel{\text{def}}{=} I \wedge ((e \wedge I) \implies wlp(C, I)) \wedge ((\neg e \wedge I) \implies P)$$



Edsger W. Dijkstra⁹

- **predicate transformer** semantics
propagate predicates on states through the program
- **weakest (liberal) precondition**
backwards, from property to prove to condition for program correctness
- calculus that can be mostly automated

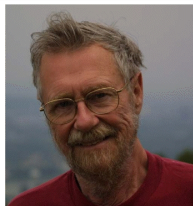
⁹ E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

A calculus of program properties

$$wlp(X := e, P) \stackrel{\text{def}}{=} P[e/X]$$

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Edsger W. Dijkstra⁹

- **predicate transformer** semantics
propagate predicates on states through the program
- **weakest (liberal) precondition**
backwards, from property to prove to condition for program correctness
- calculus that can be mostly automated, except for:
 - user annotations for inductive loop invariants
 - function annotations (modular inference)
- academic success: complex (functional) but local properties
- industry success: for simple, local properties

⁹E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

Static analysis

Principle: a program A that

- takes as input another program P
(*programs are also data!*)
- answers with “yes” if the program is safe,
“no” if it is not safe
- always answers, hopefully quickly

⇒ **proves automatically a program safe before it is run!**



Alan Turing

Limit to automation: undecidability

It is well known that termination (a useful property) is undecidable.¹⁰

In fact, all “interesting” properties are **undecidable**¹¹

⇒ A cannot exist. 😞

¹⁰ A. M. Turing. “Computability and definability”. The Journal of Symbolic Logic, vol. 2, pp. 153–163, (1937).

¹¹ H. G. Rice. “Classes of Recursively Enumerable Sets and Their Decision Problems.” Trans. Amer. Math. Soc. 74, 358-366, 1953.

Abstract interpretation

Approximate static analysis

An **approximate** static analyzer A always answers in finite time

- either **yes**: the program P is definitely safe (soundness)
- either **no**: I don't know (incompleteness)

Sufficient to prove the safety of (some) programs.

Incompleteness: A fails on infinitely many programs. . .

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Incompleteness: A fails on infinitely many programs. . .

Completeness: for any safe program P , we can design an analyzer A that proves it!

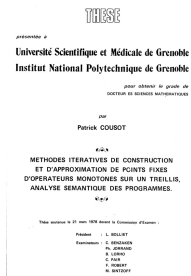
⇒ We should **adapt** the analyzer A to

- a class of programs to verify, and
- a class of safety properties to check.

Abstract interpretation



Patrick Cousot¹²



General theory of the approximation and comparison of program semantics:

- unifies many existing semantics
- allows the definition of new static analyses that are correct by construction

¹²P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

Abstract interpretation

(\mathcal{S}_0)

assume X in [0,1000];

(\mathcal{S}_1)

I := 0;

(\mathcal{S}_2)

while (\mathcal{S}_3) I < X do

(\mathcal{S}_4)

I := I + 2;

(\mathcal{S}_5)

(\mathcal{S}_6)

program

Abstract interpretation

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assume X in [0,1000];

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I := I + 2;

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(\mathcal{S}_6)

program

$\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \rightarrow \mathbb{Z})$

$\mathcal{S}_0 = \{(i, x) \mid i, x \in \mathbb{Z}\} = \top$

$\mathcal{S}_1 = \{(i, x) \in \mathcal{S}_0 \mid x \in [0, 1000]\} = F_1(\mathcal{S}_0)$

$\mathcal{S}_2 = \{(0, x) \mid \exists i, (i, x) \in \mathcal{S}_1\} = F_2(\mathcal{S}_1)$

$\mathcal{S}_3 = \mathcal{S}_2 \cup \mathcal{S}_5$

$\mathcal{S}_4 = \{(i, x) \in \mathcal{S}_3 \mid i < x\} = F_4(\mathcal{S}_3)$

$\mathcal{S}_5 = \{(i + 2, x) \mid (i, x) \in \mathcal{S}_4\} = F_5(\mathcal{S}_4)$

$\mathcal{S}_6 = \{(i, x) \in \mathcal{S}_3 \mid i \geq x\} = F_6(\mathcal{S}_3)$

semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \rightarrow \mathbb{Z})$:

- strongest invariant (and an inductive invariant)
- not computable in general
- smallest solution of a system of equations

Abstract interpretation

(S_0)

assume X in [0,1000];

(S_1)

I := 0;

(S_2)

while (S_3) I < X do

(S_4)

I := I + 2;

(S_5)

(S_6)

program

$S_i \in \mathcal{D}^\#$

$S_0^\# = \top^\#$

$S_1^\# = \llbracket \text{assume } X \in [0, 1000] \rrbracket^\#(S_0^\#)$

$S_2^\# = \llbracket I \leftarrow 0 \rrbracket^\#(S_1^\#)$

$S_3^\# = S_2^\# \cup^\# S_5^\#$

$S_4^\# = \llbracket \text{assume } I < X \rrbracket^\#(S_3^\#)$

$S_5^\# = \llbracket I \leftarrow I + 2 \rrbracket^\#(S_4^\#)$

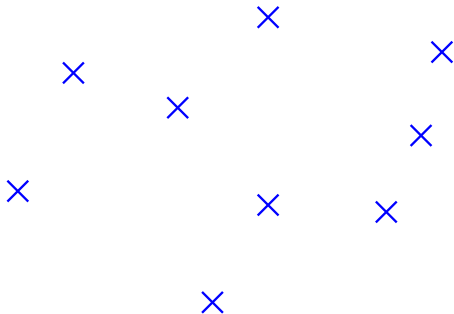
$S_6^\# = \llbracket \text{assume } I \geq X \rrbracket^\#(S_3^\#)$

semantics

Abstract semantics $S_i^\# \in \mathcal{D}^\#$:

- $\mathcal{D}^\#$ is a subset of properties of interest (approximation) with a machine representation
- $F^\# : \mathcal{D}^\# \rightarrow \mathcal{D}^\#$ over-approximates the effect of $F : \mathcal{D} \rightarrow \mathcal{D}$ in $\mathcal{D}^\#$ (with effective algorithms)

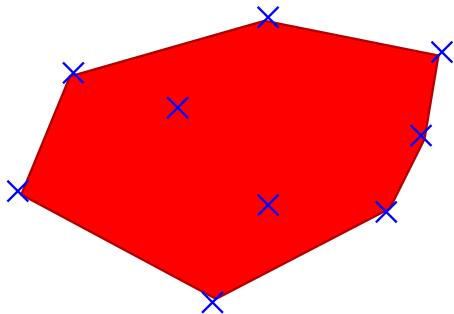
Numeric abstract domain examples



concrete sets:

$$\{(0, 3), (5.5, 0), (12, 7), \dots\} \subseteq \mathbb{R}^2$$

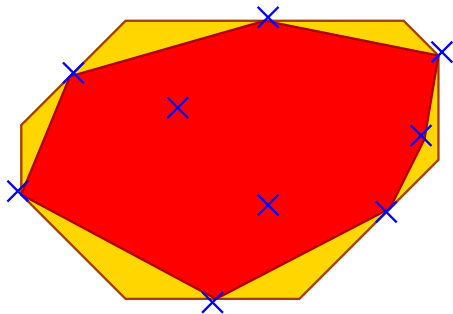
Numeric abstract domain examples



concrete sets: $\{(0, 3), (5.5, 0), (12, 7), \dots\} \subseteq \mathbb{R}^2$

abstract polyhedra: $6X + 11Y \geq 33 \wedge \dots$

Numeric abstract domain examples

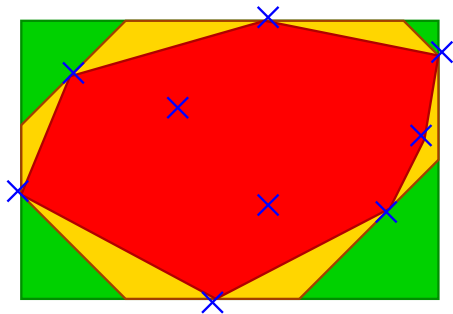


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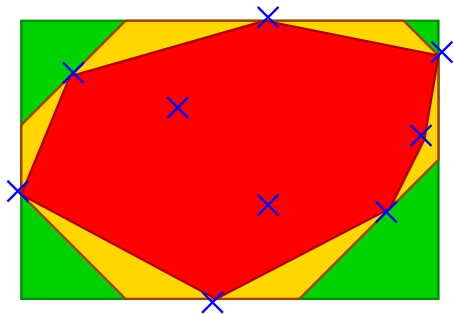
abstract octagons: $X + Y \geq 3 \wedge Y \geq 0 \wedge \dots$

Numeric abstract domain examples



- concrete sets: $\{(0, 3), (5.5, 0), (12, 7), \dots\} \subseteq \mathbb{R}^2$
- abstract polyhedra: $6X + 11Y \geq 33 \wedge \dots$
- abstract octagons: $X + Y \geq 3 \wedge Y \geq 0 \wedge \dots$
- abstract intervals: $X \in [0, 12] \wedge Y \in [0, 8]$

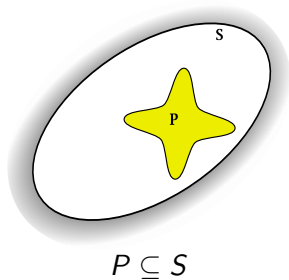
Numeric abstract domain examples



concrete sets:	$\{(0, 3), (5.5, 0), (12, 7), \dots\} \subseteq \mathbb{R}^2$	not computable
abstract polyhedra:	$6X + 11Y \geq 33 \wedge \dots$	exponential cost
abstract octagons:	$X + Y \geq 3 \wedge Y \geq 0 \wedge \dots$	cubic cost
abstract intervals:	$X \in [0, 12] \wedge Y \in [0, 8]$	linear cost

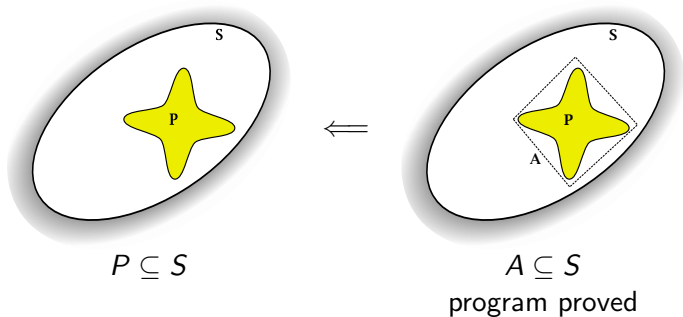
Trade-off between cost and expressiveness / precision

Soundness and false alarms



Goal : prove that a program P satisfies its specification S

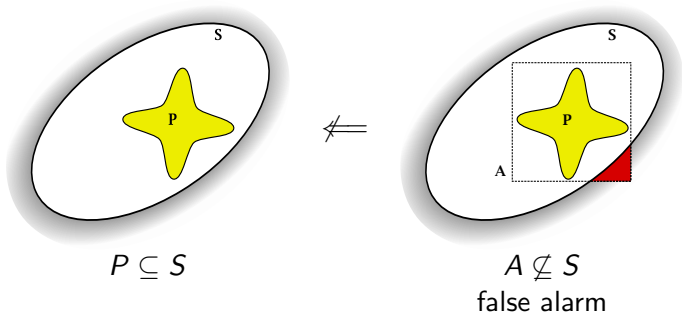
Soundness and false alarms



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A polyhedral abstraction A can prove the correctness.

Soundness and false alarms



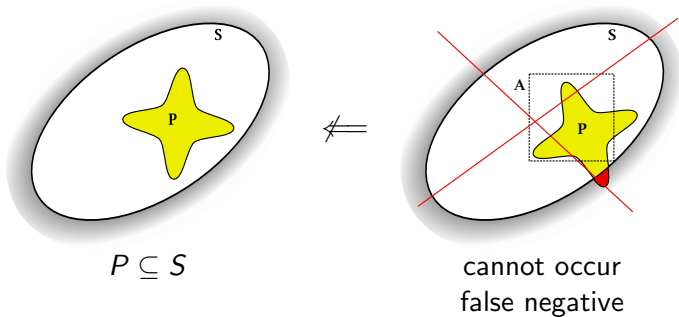
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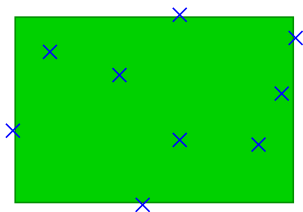
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\implies false alarm.

The analysis is sound: no false negative reported!

Abstract elements and operators



abstract semantics F^\sharp in the interval domain \mathcal{D}_i^\sharp

- $I \in \mathcal{D}_i^\sharp$ is a pair of bounds $(l, h) \in \mathbb{Z}^2$ (for each variable) representing an interval $[l, h] \subseteq \mathbb{Z}$
- $\mathbf{I} := \mathbf{I} + 2$: $(l, h) \mapsto (l+2, h+2)$
- \mathbf{U}^\sharp : $(l_1, h_1) \mathbf{U}^\sharp (l_2, h_2) = (\min(l_1, l_2), \max(h_1, h_2))$
- ...

Resolution by iteration and extrapolation

Challenge: the equation system is recursive: $\vec{S}^\# = \vec{F}^\#(\vec{S}^\#)$.

Solution: resolution by iteration: $\vec{S}^{\#0} = \emptyset^\#, \vec{S}^{\#i+1} = \vec{F}^\#(\vec{S}^{\#i})$.

e.g., $\mathcal{S}_3^\# : I \in \emptyset, I = 0, I \in [0, 2], I \in [0, 4], \dots, I \in [0, 1000]$

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Challenge: infinite or very long sequence of iterates in $\mathcal{D}^\#$

Solution: extrapolation operator ∇

e.g., $[0, 2] \nabla [0, 4] = [0, +\infty[$

- remove unstable bounds and constraints
- ensures the convergence in finite time
- **inductive** reasoning (through generalisation)

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- remove unstable bounds and constraints
- ensures the convergence in finite time
- **inductive** reasoning (through generalisation)

\implies effective solving method \longrightarrow static analyzer!

Other uses of abstract interpretation

- Analysis of dynamic memory data-structures (*shape analysis*).
- Analysis of parallel, distributed, and multi-thread programs.
- Analysis of probabilistic programs.
- Analysis of biological systems.
- Security analysis (*information flow*).
- Termination analysis.
- Cost analysis.
- Analyses to enable compiler optimisations.
- ...

A few examples of abstract interpretation tools

- **Proprietary tools**

- **PolySpace analyzer** (*MathWorks*)
run-time errors in Ada, C, C++
- **aiT** (*AbsInt*)
worst-case execution time for binary
- **Astrée** (*CNRS, ENS, INRIA, AbsInt*)
run-time errors in embedded C, with an emphasis on validation
- **Sparrow** (*Seoul National University*)
run-time errors in C
- **Julia** (*University of Verona*)
analysis of Java and Android

- **Open-source tools**

- **Frama-C** (*CEA LIST, INRIA, TrustInSoft*)
run-time errors in C software, also has a commercial version
- **Code Contracts Static Checker** (*Microsoft Research*)
static checking and inference of .NET contracts

The Astrée static analyzer

The screenshot displays the Astrée static analyzer interface. The main window is titled "Astrée" and shows two panes for code analysis. The left pane, "Analyzed file: /invalid/path/scenarios.c", shows the analyzed code with line numbers 24 to 49. The right pane, "Original source: C:/Pr...ples/scenarios/src/scenarios.c", shows the original source code with line numbers 37 to 61. The code includes comments and assignments, such as `z = {short}((unsigned short)vx + (unsigned short)vy);` and `ASTREE_assert((-2 <= z && z <= 2));`. The interface also features a sidebar with "Local settings" and "Analysis options", a status bar at the bottom, and a table of errors and alarms.

Errors	Alarms	Not analyzed	Coverage	Files
2 (2)	5 (5)	0	100%	scenarios.c

Summary | Warnings | Log | Graph | Watch | Messages

Connected to localhost:1059 as anonymous@ABSINT-VMWARE

Analyseur statique de programmes temps-réels embarqués

(static analyzer for real-time embedded software)

- developed at **ENS**
 - | B. Blanchet, P. Cousot, R. Cousot, J. Feret,
 - | L. Mauborgne, D. Monniaux, A. Miné, X. Rival
- industrialized and made commercially available by **AbsInt**



Astrée

www.astree.ens.fr



AbsInt

www.absint.com

The Astrée static analyzer

Specialized:

- for the analysis of **run-time errors**
(arithmetic overflows, array overflows, divisions by 0, etc.)
- on embedded critical **C** software
(no dynamic memory allocation, no recursivity)
- in particular on **control / command** software
(reactive programs, intensive floating-point computations)
- intended for **validation**
(analysis does not miss any error and tries to minimise false alarms)

Approximately **40 abstract domains** are used **at the same time**:

- numeric domains (intervals, octagons, ellipsoids, etc.)
- boolean domains
- domains expressing properties on the history of computations

Astrée applications



Airbus A340-300 (2003)



Airbus A380 (2004)



(model of) ESA ATV (2008)

- size: from 70 000 to 860 000 lines of C
- analysis time: from 45mn to \simeq 40h
- 0 alarm: **proof of absence of run-time error**