Introduction

MPRI 2–6: Abstract Interpretation,
application to verification and static analysis

Antoine Miné

Year 2019–2020

Course 00
11 September 2019

Formal Verification: Motivation

The cost of software failure

- Patriot MIM-104 failure, 25 February 1991
  (death of 28 soldiers[1])
- Ariane 5 failure, 4 June 1996
  (cost estimated at more than 370 000 000 US$[2])
- Toyota electronic throttle control system failure, 2005
  (at least 89 death[3])
- Heartbleed bug in OpenSSL, April 2014
- the economic cost of software bugs is tremendous[4]
- ...

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1 R. Skeel. "Roundoff Error and the Patriot Missile". SIAM News, volume 25, nr 4.

2 M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.

3 CBSNews. Toyota "Unintended Acceleration" Has Killed 89. 20 March 2014.

4 NIST. Software errors cost U.S. economy $59.5 billion annually. Tech. report, NIST Planning Report, 2002.

A classic example: Ariane 5, Flight 501

Cause: software error[5]

- arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types[6]

```
P_M_DERIVE(T_ALG.E_BH) :=
  UC_16S_EN_16NS (TDB.T_ENTIER_16S
    ((1.0/C_M_LSB_BH) * G_M_INFO_DERIVE(T_ALG.E_BH)));
```

- software exception not caught
  $\Longrightarrow$ computer switched off
- all backup computers run the same software
  $\Longrightarrow$ all computers switched off, no guidance
  $\Longrightarrow$ rocket self-destructs

A "simple" error...

[5] J.-L. Lions et al., Ariane 501 Inquiry Board report.

[6] J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d'informatique de l'ENS, 2010.

How can we avoid such failures?

- Choose a safe programming language.

  C (low level) / Ada, Java, OCaml (high level)

  yet, Ariane 5 software is written in Ada

- Carefully design the software.

  many software development methods exist

  yet, critical embedded software follow strict development processes

- Test the software extensively.

  yet, the erroneous code was well tested. . . on Ariane 4

$\Longrightarrow$ **not sufficient!**

How can we avoid such failures?

- Choose a safe programming language.

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  yet, Ariane 5 software is written in Ada

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  many software development methods exist

  yet, critical embedded software follow strict development processes

- Test the software extensively.

  yet, the erroneous code was well tested... on Ariane 4

$\Longrightarrow$ **not sufficient!**

We should use **formal methods.**

provide rigorous, mathematical insurance of correctness
may not prove everything, but give a precise notion of what is proved

This case triggered the first large scale static code analysis
(PolySpace Verifier, using abstract interpretation)

Verification: compromises

Undecidability: correctness properties are undecidable!

cannot build a program that automatically and precisely separates all correct programs from all incorrect ones

Compromises:

lose automation, completeness, soundness, or generality

- **Test**: complete and automatic, but unsound
- **Theorem proving**
  - proof essentially manual, but checked automatically
  - powerful, but very steep learning curve
- **Deductive methods**
  - automated proofs for some logic fragments (SAT, SMT)
  - still requires program annotations (contracts, invariants)
- **Model checking**
  - check a (often hand-crafted) model of the program
  - finite or regular models, expressive properties (LTL)
  - automatic and complete (wrt. model)
- **Static analysis** (next slide)

Verification by static analysis

source

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    if (t[i]) break;
  }
  return t[i];
}
```

$\Longrightarrow$

analysis result

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    // 0 ≤ i < n
    if (t[i]) break;          ✓
  }
  // 0 ≤ i ≤ n ∨ n < 0
  return t[i];                ✗
}
```

- work directly on the source code
- infer properties on program executions
- automatically (cost effective)
- construct dynamically a semantic abstraction of the program
- deduce program correctness or raise alarms
  (implicit specification: absence of RTE; or user-defined properties: contracts)
- with approximations (incomplete: efficient, but possible false alarms)
- soundly (no false positive)

Verification in practice: The example of avionics software

Critical avionics software is subject to certification:

- more than half the development cost
- regulated by international standards (DO-178B, DO-178C)
- mostly based on massive test campaigns & intellectual reviews

Current trend:

use of **formal methods** now acknowledged (DO-178C, DO-333)

- at the binary level, to replace testing
- at the source level, to replace intellectual reviews
- at the source level, to replace testing
  provided the correspondence with the binary is also certified

$\Longrightarrow$ **formal methods can improve cost-effectiveness!**

Caveat: **soundness** is required by DO

Verification in practice: Formal verification at Airbus

Program proofs: deductive methods

- functional properties of small sequential C codes
- replace unit testing
- not fully automatic
- **Caveat** / **Frama-C** tool (CEA)

Sound static analysis:

- fully automated on large applications, non functional properties
- worst-case execution time and stack usage, on binary
  aiT, **StackAnalyzer** (AbsInt)
- absence of run-time error, on sequential C code
  Astrée analyzer (AbsInt)

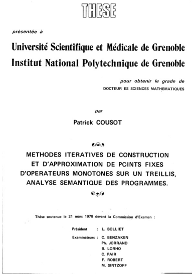
Certified compilation:

- allows source-level analysis to certify sequential binary code
- **CompCert** C compiler, certified in **Coq** (INRIA)

Overview of abstract interpretation

Abstract interpretation

Patrick Cousot[7]



THESE

présentée à

Université Scientifique et Médicale de Grenoble
Institut National Polytechnique de Grenoble

pour obtenir le grade de
DOCTEUR ES SCIENCES MATHEMATIQUES

par

Patrick COUSOT

METHODES ITERATIVES DE CONSTRUCTION
ET D'APPROXIMATION DE PCINTS FIXES
D'OPERATEURS MONOTONES SUR UN TREILLIS,
ANALYSE SEMANTIQUE DES PROGRAMMES.

Thèse soutenue le 21 mars 1978 devant la Commission d'Examen :

Président : L. BOLLIET
Examinateurs : C. BENZAKEN
Ph. JORRAND
B. LORHO
C. PAIR
F. ROBERT
M. SINTZOFF

General theory of the approximation and comparison of program semantics:

- unifies existing semantics
- guides the design of static analyses that are correct by construction

[7] P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

Concrete collecting semantics

$(\mathcal{S}_0)$

```
assume X in [0,1000];
```

$(\mathcal{S}_1)$

```
I := 0;
```

$(\mathcal{S}_2)$

while $(\mathcal{S}_3)$ `I < X do`

  $(\mathcal{S}_4)$

```
    I := I + 2;
```

  $(\mathcal{S}_5)$

$(\mathcal{S}_6)$

program

Concrete collecting semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$$\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I}, \mathtt{X}\} \to \mathbb{Z})$$

$$\begin{array}{lll}
\mathcal{S}_0 &= \{\, (i,x) \mid i,x \in \mathbb{Z} \,\} & = \top \\
\mathcal{S}_1 &= \{\, (i,x) \in \mathcal{S}_0 \mid x \in [0,1000] \,\} & = F_1(\mathcal{S}_0) \\
\mathcal{S}_2 &= \{\, (0,x) \mid \exists i,\ (i,x) \in \mathcal{S}_1 \,\} & = F_2(\mathcal{S}_1) \\
\mathcal{S}_3 &= \mathcal{S}_2 \cup \mathcal{S}_5 & \\
\mathcal{S}_4 &= \{\, (i,x) \in \mathcal{S}_3 \mid i < x \,\} & = F_4(\mathcal{S}_3) \\
\mathcal{S}_5 &= \{\, (i+2,x) \mid (i,x) \in \mathcal{S}_4 \,\} & = F_5(\mathcal{S}_4) \\
\mathcal{S}_6 &= \{\, (i,x) \in \mathcal{S}_3 \mid i \geq x \,\} & = F_6(\mathcal{S}_3)
\end{array}$$

semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I}, \mathtt{X}\} \to \mathbb{Z})$:

- strongest program properties (inductive invariants)
- set of reachable environments, at each program point
- smallest solution of a system of equations
- well-defined solution, but not computable in general

Abstracting

Principle: be tractable by reasoning at an abstract level

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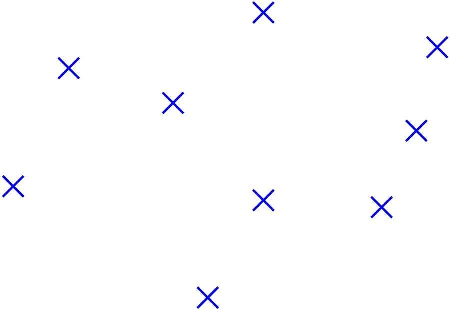
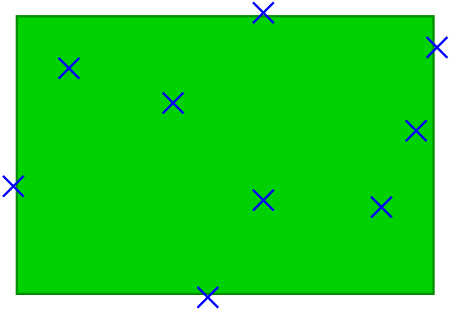


concrete executions : $\{(0,3),(5.5,0),(12,7),\ldots\}$ (not computable)

Abstracting

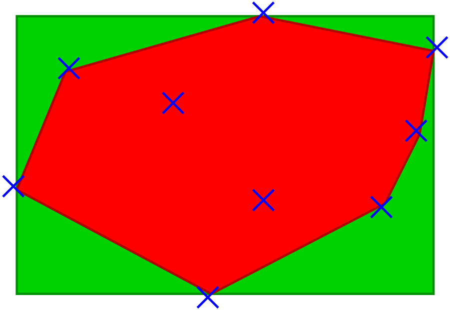
Principle: be tractable by reasoning at an abstract level



concrete executions :	$\{(0,3),(5.5,0),(12,7),\ldots\}$	(not computable)
box domain :	$X \in [0,12] \wedge Y \in [0,8]$	(linear cost)

Abstracting

Principle: be tractable by reasoning at an abstract level



concrete executions : $\{(0,3),(5.5,0),(12,7),\ldots\}$ (not computable)

box domain : $X \in [0,12] \wedge Y \in [0,8]$ (linear cost)

polyhedra domain : $6X + 11Y \geq 33 \wedge \cdots$ (exponential cost)

many abstractions: trade-off cost vs. precision and expressiveness

From concrete to abstract semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$$\mathcal{S}_i \in \mathcal{D} \stackrel{\text{def}}{=} \mathcal{P}(\{I, X\} \to \mathbb{Z})$$

$$\begin{aligned}
\mathcal{S}_0 &= \{\, (i,x) \mid i,x \in \mathbb{Z} \,\} \\
\mathcal{S}_1 &= [\![X \in [0,1000]]\!] \, (\mathcal{S}_0) \\
\mathcal{S}_2 &= [\![I \leftarrow 0]\!] \, (\mathcal{S}_1) \\
\mathcal{S}_3 &= \mathcal{S}_2 \cup \mathcal{S}_5 \\
\mathcal{S}_4 &= [\![I < X]\!] \, (\mathcal{S}_3) \\
\mathcal{S}_5 &= [\![I \leftarrow I + 2]\!] \, (\mathcal{S}_4) \\
\mathcal{S}_6 &= [\![I \geq X]\!] \, (\mathcal{S}_3)
\end{aligned}$$

concrete semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \to \mathbb{Z})$:

- $[\![X \in [0,1000]]\!]$, $[\![I \leftarrow 0]\!]$, etc. are transfer functions
- strongest program properties
- set of reachable environments, at each program point
- not computable in general

From concrete to abstract semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$$\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$$

$$\begin{aligned}
\mathcal{S}_0^\sharp &= \top^\sharp \\
\mathcal{S}_1^\sharp &= [\![X \in [0,1000]]\!]^\sharp(\mathcal{S}_0^\sharp) \\
\mathcal{S}_2^\sharp &= [\![I \leftarrow 0]\!]^\sharp(\mathcal{S}_1^\sharp) \\
\mathcal{S}_3^\sharp &= S_2^\sharp \cup^\sharp \mathcal{S}_5^\sharp \\
\mathcal{S}_4^\sharp &= [\![I < X]\!]^\sharp(\mathcal{S}_3^\sharp) \\
\mathcal{S}_5^\sharp &= [\![I \leftarrow I + 2]\!]^\sharp(\mathcal{S}_4^\sharp) \\
\mathcal{S}_6^\sharp &= [\![I \geq X]\!]^\sharp(\mathcal{S}_3^\sharp)
\end{aligned}$$

abstract semantics

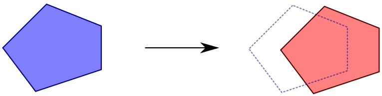
Abstract semantics $\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$:

- $\mathcal{D}^\sharp$ is a subset of properties of interest
  semantic choice + a machine representation
- $F^\sharp : \mathcal{D}^\sharp \to \mathcal{D}^\sharp$ over-approximates the effect of $F : \mathcal{D} \to \mathcal{D}$ in $\mathcal{D}^\sharp$
  with effective algorithms

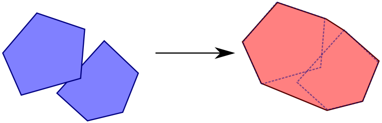
Abstract operator examples

In the polyhedra domain:

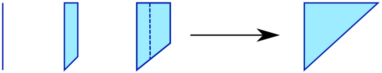
- Abstract assignment
  $[\![X \leftarrow X + 1]\!]^\sharp$
  translation (exact)



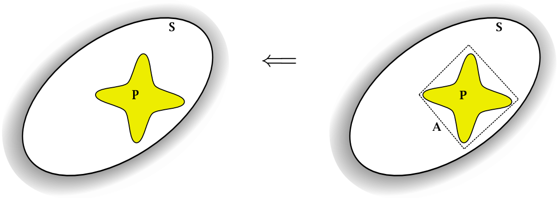
- Abstract union
  $\cup^\sharp$
  convex hull (approximate)



- Solving the equation system
  by iteration
  using extrapolation to terminate



Soundness and false alarms



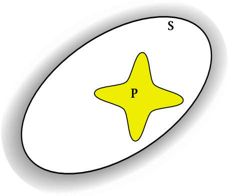
$P \subseteq S$ $\qquad$ $A \subseteq S$
program proved

Goal : prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

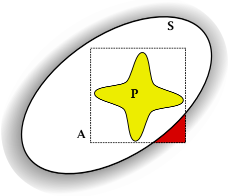
A polyhedral abstraction A can prove the correctness

Soundness and false alarms



$P \subseteq S$

$\not\Leftarrow$



$A \not\subseteq S$
false alarm

Goal : prove that a program P satisfies its specification S

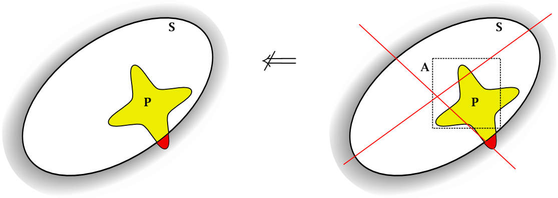
We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

Soundness and false alarms



$P \not\subseteq S$

$A \subseteq S$
false negative
cannot occur

Goal : prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

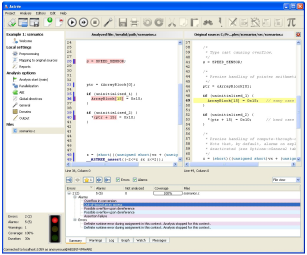
The analaysis is sound: no false negative reported!

Example static analyzer: Astrée

Astrée: developed at ENS & INRIA by P. Cousot & al.

- analyzes embedded critical C software
  subset of C, no memory allocation, no recursivity → simpler semantics
- checks for run-time errors
  arithmetic overflows, array overflows, divisions by 0, pointer errors, etc. → non-functional
- specialized for control / command software
  with zero false alarm goal
  application domain specific abstractions

Airbus A380



2001–2004: academic success
proof of absence of RTE
on flight command

2009: industrialization

AbsInt

Example static analyzer: Infer.AI at Facebook

Infer: http://fbinfer.com/

- developed at Facebook (team formerly at Monoidics)
- Infer.AI is an analysis framework based on abstract interpretation
- open-source since 2015
- analyzes Java, C, C++, and Objective-C
- checks ThreadSafety (Java), Initalisation Order (C++), etc.
- modular, bottom-up interprocedural analysis
- targets the analysis of merge requests (small bits at a time)
- favors speed over soundness

  pragmatic choices, based on "what engineers want"
  no requirements for certification, unlike the avionics industry
- used in production

Course organisation

Teaching team

Cezara Drăgoi

Jérôme Feret

Antoine Miné

Xavier Rival

Syllabus and exams

https://www-apr.lip6.fr/~mine/enseignement/mpri/2019-2020

Visit regularly for:

- latest information on course dates
- course material
- course assignments
- internship proposals

Exams:

- 50%: written mid-term exam (3h)
- 50%: oral final exam
  (read a scientific article, present it, answer questions)

Course material

Links available on the web-page:

- main material: slides
- course notes

  cover mainly foundations and numeric abstract domains based on:

  A. Miné. *Tutorial on Static Inference of Numeric Invariants by Abstract Interpretation*. In Foundations and Trends in Programming Languages, 4(3–4), 120–372. Now Publishers.

- recommended reading on theory and applications:

  J. Bertrane, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, X. Rival. *Static analysis and verification of aerospace software by abstract interpretation*. In Foundations and Trends in Programming Languages, 2(2–3), 71–190, 2015. Now Publishers.

Course assignments (self-evaluated)

On the web page, **highly recommended** homework

- exercises: prove a theorem, solve a former exam, etc.
- reading assignments: an article related to the course
- experiments: use a tool

Also:

- previous exams, with correction
- example programming project
  (abstract interpreter for a toy language in OCaml)

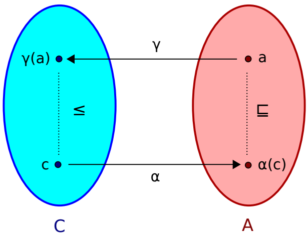
Principle: self-evaluation

No credit.
Not evaluated by the teacher.

Course plan (1/8)

Foundations of abstract interpretation: (courses 1 & 2)

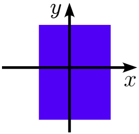
- mathematical background: order theory and fixpoints
- formalization of abstraction, soundness
- program semantics and program properties
- hierarchy of collecting semantics



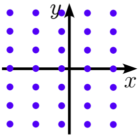
Course plan (2/8)

Basic bricks of abstraction: numerical domains (courses 3, 4 & 15)

simple domains

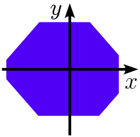


Intervals
$x \in [a, b]$

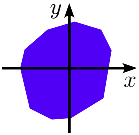


Congruences
$x \in a\mathbb{Z} + b$

relational domains

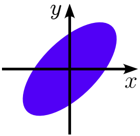


Octagons
$\pm x \pm y \leq c$

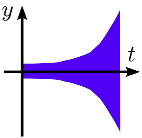


Polyhedra
$\sum_i \alpha_i x_i \leq \beta$

specific domains



Ellipsoids
digital filters

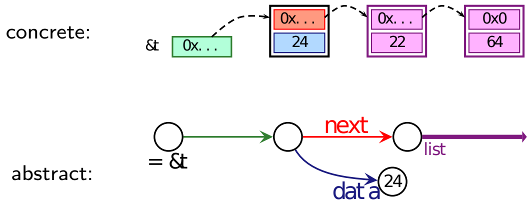


Exponentials
rounding errors

Course plan (3/8)

Basic bricks of abstraction: memory abstractions (courses 7 & 11)

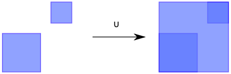
- beyond numeric: reason on arrays, lists, trees, graphs, ...
- challenges: variety of structures, destructive updates
- logical tools:
  - separation logics (a logic tailored for describing memory)
  - parametric three valued logics (representing arbitrary graphs)
- abstract domains based on these logics



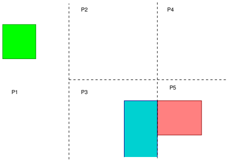
Course plan (4/8)

Basic bricks of abstraction: partitioning abstractions (course 10)

- most abstract domains are not distributive
  $\Longrightarrow$ reasoning over disjunctions loses precision
- first solution: add disjunctions to any abstract domain
  $\Longrightarrow$ expressive but costly
- second solution: partitioning
  conjunctions of implications as logical predicates
  (partitioning may be based on many semantic criteria)



loss of precision



partitioning

Course plan (5/8)

Analyses: analysis of concurrent data-structures (courses 8 & 9)

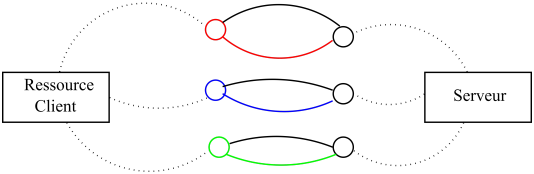
- abstract domains to reason about relations between data structures
- thread-modular abstractions
- program logic combing rely-guarantee and separation logic
- concurrent data-structure verification
  (reduction to state reachability provable by the abstract domains)

Course plan (6/8)

Analyses: analysis of mobile systems (courses 12 & 13)

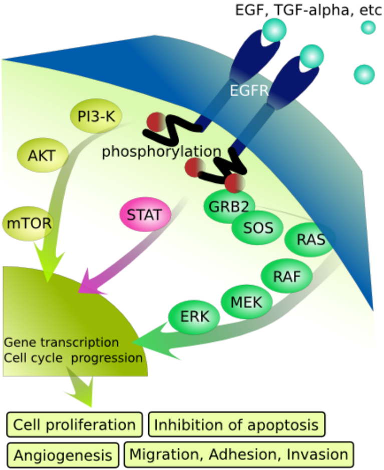
- dynamic creation of components and links
- analyze the links between components
  - distinguish between recursive components
  - abstractions as sets of words
- bound the number of components

  using numeric relations



Course plan (7/8)

Analyses: abstractions of signaling pathways (courses 5 & 6)

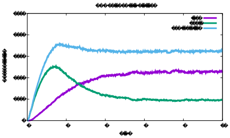


[Eikuch, 2007]

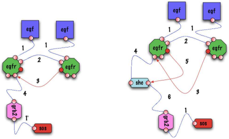
Course plan (7/8)

Analyses: abstractions of signaling pathways (courses 5 & 6)

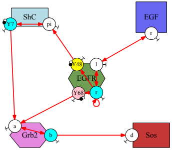
abstractions offer different perspectives on models



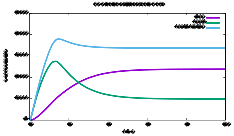
concrete semantics



causal traces



information f ow



exact projection
of the ODE semantics

Course plan (8/8)

Analyses: static analysis for security (course 16)

- challenge: security properties are diverse
  from information leakage to unwanted execution of malicious code
  and more complex than safety and liveness
- the framework of hyperproperties can express security
- apply abstract interpretation to reason over non-interference

Internship proposals

Possibility of Master 2 internships at ENS or Sorbonne Université.

Example topics:

- Static analysis of smart contracts
- Semantic input data usage analysis
- Algorithmic fairness analysis of neural networks
- Counter-example generation through backward under-approximations
- Static analysis for lock-free data structures
- Static analysis for consensus algorithms
- ...

Formal proposals will be available on the course page
also: discuss with your teachers for tailor-made subjects.