Introduction

MPRI 2–6: Abstract Interpretation,
application to verification and static analysis

Antoine Miné

Year 2022–2023

Course 0
19 September 2022

Formal Verification: Motivation

Historic example: Ariane 5, Flight 501

Maiden flight of the Ariane 5 Launcher, 4 June 1996.
Cost of failure estimated at more than 370 000 000 US\$[1]

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[1] M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.

Cause of Ariane 5 failure

Cause: software error[2]

- arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types[3]

```
P_M_DERIVE(T_ALG.E_BH) :=
  UC_16S_EN_16NS (TDB.T_ENTIER_16S
    ((1.0/C_M_LSB_BH) * G_M_INFO_DERIVE(T_ALG.E_BH)));
```

- software exception not caught
  $\Longrightarrow$ computer switched off
- all backup computers run the same software
  $\Longrightarrow$ all computers switched off, no guidance
  $\Longrightarrow$ rocket self-destructs

A “simple” error…

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[2] J.-L. Lions et al., Ariane 501 Inquiry Board report.

[3] J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d’informatique de l’ENS, 2010.

How can we avoid such failures?

- Choose a safe programming language.

  C (low level) / Ada, Java, OCaml (high level)

  yet, Ariane 5 software is written in Ada

- Carefully design the software.

  many software development methods exist

  yet, critical embedded software follow strict development processes

- Test the software extensively.

  yet, the erroneous code was well tested... on Ariane 4

$\Longrightarrow$ **not sufficient!**

How can we avoid such failures?

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- Carefully design the software.

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  yet, critical embedded software follow strict development processes

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  yet, the erroneous code was well tested. . . on Ariane 4

  $\Longrightarrow$ **not sufficient!**

We should use **formal methods.**

provide rigorous, mathematical insurance of correctness
may not prove everything, but give a precise notion of what is proved

This case triggered the first large scale static code analysis

PolySpace Verifier, using abstract interpretation

Verification: compromises

Undecidability: correctness properties are undecidable!

no program can automatically and precisely separates all correct programs from all incorrect ones

Compromises: lose automation, or completeness, or soundness, or generality

- **Test, symbolic execution**: complete and automatic, but unsound
- **Theorem proving**
  - proof essentially manual, but checked automatically
  - powerful, but very steep learning curve and large effort required
- **Deductive methods**
  - automated proofs for some logic fragments (SAT, SMT)
  - still requires some program annotations (contracts, invariants)
- **Model checking**
  - check a (often hand-crafted) model of the program
  - finite or regular models, expressive properties (LTL)
  - automatic and complete (wrt. model)
- **Static analysis** (next slide)

Verification by static analysis

source

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    if (t[i]) break;
  }
  return t[i];
}
```

⟹

analysis result

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    // 0 ≤ i < n
    if (t[i]) break;                  ✓
  }
  // 0 ≤ i ≤ n ∨ n < 0
  return t[i];                        ✗
}
```

- work directly on the source code
- infer properties on program executions
- automatically (cost effective)
- by constructing dynamically a semantic abstraction of the program
- to deduce program correctness, or raise alarms if it cannot

  implicit specification: absence of RTE; or (simple) user-defined properties: contracts
- with approximations (incomplete: efficient, but possible false alarms)
- soundly (no false positive)

Verification in practice: Example of avionics software

Critical avionics software is subject to certification:

- 70% of the development cost (in 2015)
- regulated by international standards (DO-178)
- mostly based on massive test campaigns & intellectual reviews

Current trend:

use of **formal methods** now acknowledged (DO-178C, DO-333)

- at the binary level, to replace testing
- at the source level, to replace intellectual reviews
- at the source level, to replace testing
  provided that the correspondence with the binary is also certified

$\Longrightarrow$ **formal methods can improve cost-effectiveness!**

Caveat: **soundness** is required by DO standards

Verification in practice: Formal verification at Airbus

Program proofs: deductive methods

- functional properties of small sequential C codes
- replace unit testing
- not fully automatic
- **Caveat / Frama-C** tool (CEA)

Sound static analysis:

- fully automated on large applications, non functional properties
- worst-case execution time and stack usage, on binary
  aiT, **StackAnalyzer** (AbsInt)
- absence of run-time error, on sequential C code
  Astrée analyzer (AbsInt)

Certified compilation:

- allows source-level analysis to certify sequential binary code
- **CompCert** C compiler, certified in **Coq** (INRIA)

Another example bug: Heartbleed

Vulnerability in OpenSSL cryptographic library
all versions from 2012 to 2014

OpenSSL is used by 66% of WEB servers for `https`
(also: email encryption, VPN, etc.)

Cause: buffer overflow in "heartbeat" protocol.

Consequence:[4]

- leak of private information, such as private keys
- no way to actually know what has been extracted
  $\Longrightarrow$ need to renew all keys after correcting the bug!
- very high economic cost!

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[4] `http://heartbleed.com`

Improving software quality

Recent study from Consortium for Information & Software Quality:[5]

- $607 billions spent finding and fixing bugs
- $1.56 trillon cost for software failure
- just for 2020, just for the US!

$\Longrightarrow$ even non-critical domains could use formal methods!

Challenges:

- keep up with scalability on critical software
- go beyond critical software (larger, more complex)
- more complex languages and programming models (C++, JavaScript, Python, ...)
- go beyond absence of run-time errors and towards functional properties
- while still being sound!

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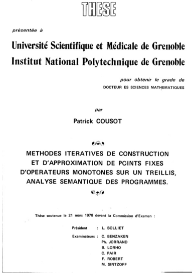
[5] Herb Krasner. The cost of poor software quality in the US: A 2020 report. `https://www.it-cisq.org/pdf/CPSQ-2020-report.pdf`, 2021. Accessed: 2021-08.

Overview of abstract interpretation

Abstract interpretation

Patrick Cousot[6]



THESE

présentée à

Université Scientifique et Médicale de Grenoble
Institut National Polytechnique de Grenoble

pour obtenir le grade de
DOCTEUR ES SCIENCES MATHEMATIQUES

par

Patrick COUSOT

⁂

METHODES ITERATIVES DE CONSTRUCTION
ET D'APPROXIMATION DE POINTS FIXES
D'OPERATEURS MONOTONES SUR UN TREILLIS,
ANALYSE SEMANTIQUE DES PROGRAMMES.

⁂

Thèse soutenue le 21 mars 1978 devant la Commission d'Examen :

Président : L. BOLLIET
Examinateurs : C. BENZAKEN
Ph. JORRAND
B. LORHO
C. PAIR
F. ROBERT
M. SINTZOFF

General theory of the approximation and comparison of program semantics:

- unifies existing semantics
- guides the design of static analyses
  that are correct by construction

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[6] P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

Concrete collecting semantics

$(\mathcal{S}_0)$

```
assume X in [0,1000];
```

$(\mathcal{S}_1)$

```
I := 0;
```

$(\mathcal{S}_2)$

`while` $(\mathcal{S}_3)$ `I < X do`

$(\mathcal{S}_4)$

```
    I := I + 2;
```

$(\mathcal{S}_5)$

$(\mathcal{S}_6)$

program

Concrete collecting semantics

```
$(\mathcal{S}_0)$
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I := 0;
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while $(\mathcal{S}_3)$ I < X do
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    I := I + 2;
    $(\mathcal{S}_5)$
$(\mathcal{S}_6)$
```

program

$$\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I},\mathtt{X}\} \to \mathbb{Z})$$

$$\begin{array}{lll}
\mathcal{S}_0 = \{\, (i,x) \mid i,x \in \mathbb{Z} \,\} & = \top \\
\mathcal{S}_1 = \{\, (i,x) \in \mathcal{S}_0 \mid x \in [0,1000] \,\} & = F_1(\mathcal{S}_0) \\
\mathcal{S}_2 = \{\, (0,x) \mid \exists i,\ (i,x) \in \mathcal{S}_1 \,\} & = F_2(\mathcal{S}_1) \\
\mathcal{S}_3 = S_2 \cup \mathcal{S}_5 & \\
\mathcal{S}_4 = \{\, (i,x) \in \mathcal{S}_3 \mid i < x \,\} & = F_4(\mathcal{S}_3) \\
\mathcal{S}_5 = \{\, (i+2,x) \mid (i,x) \in \mathcal{S}_4 \,\} & = F_5(\mathcal{S}_4) \\
\mathcal{S}_6 = \{\, (i,x) \in \mathcal{S}_3 \mid i \geq x \,\} & = F_6(\mathcal{S}_3)
\end{array}$$

semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I},\mathtt{X}\} \to \mathbb{Z})$:

- strongest program properties (inductive invariants)
- set of reachable environments, at each program point
- smallest solution of a system of equations
- well-defined solution, but not computable in general

Abstracting

Principle: be tractable by reasoning at an abstract level

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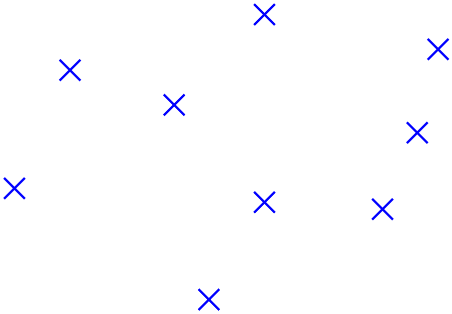
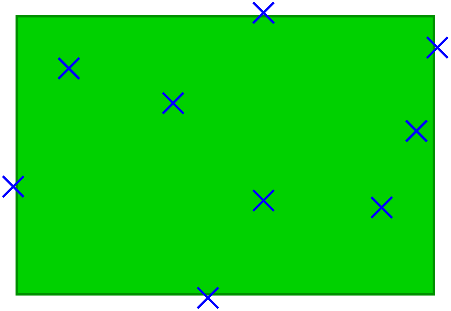


concrete executions : $\{(0,3),(5.5,0),(12,7),\ldots\}$ (not computable)

Abstracting

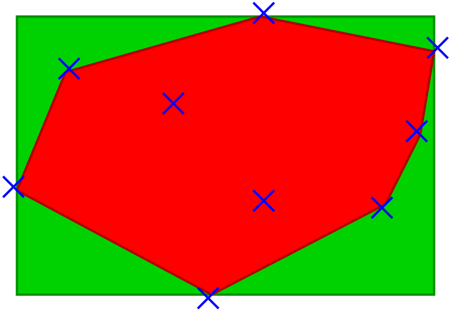
Principle: be tractable by reasoning at an abstract level



concrete executions :	$\{(0,3),(5.5,0),(12,7),\ldots\}$	(not computable)
box domain :	$X \in [0,12] \wedge Y \in [0,8]$	(linear cost)

Abstracting

Principle: be tractable by reasoning at an abstract level



concrete executions :	$\{(0,3),(5.5,0),(12,7),\ldots\}$	(not computable)
box domain :	$X \in [0,12] \wedge Y \in [0,8]$	(linear cost)
polyhedra domain :	$6X + 11Y \geq 33 \wedge \cdots$	(exponential cost)

many abstractions: trade-off cost vs. precision and expressiveness

From concrete to abstract semantics

$(\mathcal{S}_0)$
`assume X in [0,1000];`
$(\mathcal{S}_1)$
`I := 0;`
$(\mathcal{S}_2)$
`while` $(\mathcal{S}_3)$ `I < X do`
    $(\mathcal{S}_4)$
    `I := I + 2;`
    $(\mathcal{S}_5)$
$(\mathcal{S}_6)$

program

$$\mathcal{S}_i \in \mathcal{D} \overset{\text{def}}{=} \mathcal{P}(\{I, X\} \to \mathbb{Z})$$

$$\begin{aligned}
\mathcal{S}_0 &= \{\, (i,x) \mid i,x \in \mathbb{Z} \,\} \\
\mathcal{S}_1 &= [\![\, X \in [0,1000] \,]\!]\, (\mathcal{S}_0) \\
\mathcal{S}_2 &= [\![\, I \leftarrow 0 \,]\!]\, (\mathcal{S}_1) \\
\mathcal{S}_3 &= \mathcal{S}_2 \cup \mathcal{S}_5 \\
\mathcal{S}_4 &= [\![\, I < X \,]\!]\, (\mathcal{S}_3) \\
\mathcal{S}_5 &= [\![\, I \leftarrow I + 2 \,]\!]\, (\mathcal{S}_4) \\
\mathcal{S}_6 &= [\![\, I \geq X \,]\!]\, (\mathcal{S}_3)
\end{aligned}$$

concrete semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \to \mathbb{Z})$:

- $[\![\, X \in [0,1000] \,]\!]$, $[\![\, I \leftarrow 0 \,]\!]$, etc. are transfer functions
- strongest program properties
- set of reachable environments, at each program point
- not computable in general

From concrete to abstract semantics

$(\mathcal{S}_0)$
`assume X in [0,1000];`
$(\mathcal{S}_1)$
`I := 0;`
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`while` $(\mathcal{S}_3)$ `I < X do`
    $(\mathcal{S}_4)$
    `I := I + 2;`
    $(\mathcal{S}_5)$
$(\mathcal{S}_6)$

program

$$\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$$

$$\begin{aligned}
\mathcal{S}_0^\sharp &= \top^\sharp \\
\mathcal{S}_1^\sharp &= [\![\, X \in [0,1000] \,]\!]^\sharp(\mathcal{S}_0^\sharp) \\
\mathcal{S}_2^\sharp &= [\![\, I \leftarrow 0 \,]\!]^\sharp(\mathcal{S}_1^\sharp) \\
\mathcal{S}_3^\sharp &= S_2^\sharp \;\cup^\sharp\; \mathcal{S}_5^\sharp \\
\mathcal{S}_4^\sharp &= [\![\, I < X \,]\!]^\sharp(\mathcal{S}_3^\sharp) \\
\mathcal{S}_5^\sharp &= [\![\, I \leftarrow I + 2 \,]\!]^\sharp(\mathcal{S}_4^\sharp) \\
\mathcal{S}_6^\sharp &= [\![\, I \geq X \,]\!]^\sharp(\mathcal{S}_3^\sharp)
\end{aligned}$$

abstract semantics

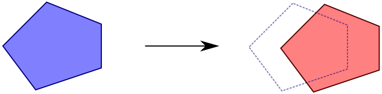
Abstract semantics $\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$:

- $\mathcal{D}^\sharp$ is a subset of properties of interest
  semantic choice + machine representation
- $F^\sharp : \mathcal{D}^\sharp \to \mathcal{D}^\sharp$ over-approximates the effect of $F : \mathcal{D} \to \mathcal{D}$ in $\mathcal{D}^\sharp$
  abstract operators proved sound + effective algorithms

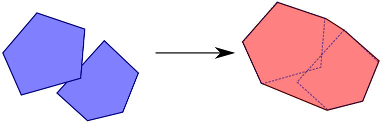
Abstract operator examples

In the polyhedra domain:

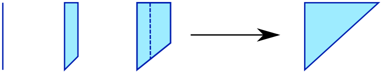
- Abstract assignment
  $[\![\, X \leftarrow X + 1 \,]\!]^\sharp$
  translation (exact)



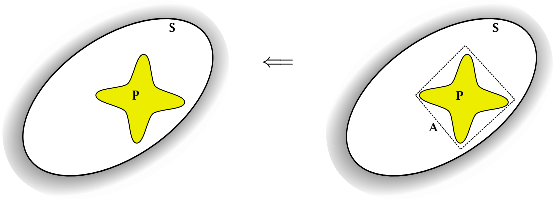
- Abstract union
  $\cup^\sharp$
  convex hull (approximate)



- Solving the equation system
  by iteration
  using extrapolation to terminate



Soundness and false alarms



$P \subseteq S$

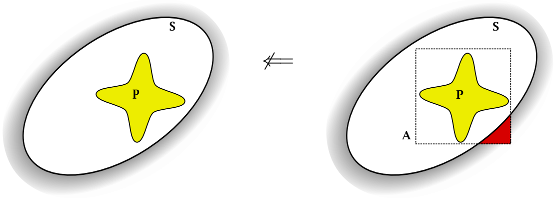
$A \subseteq S$
program proved

Goal: prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

Soundness and false alarms



$P \subseteq S$

$A \not\subseteq S$
false alarm

Goal: prove that a program P satisfies its specification S

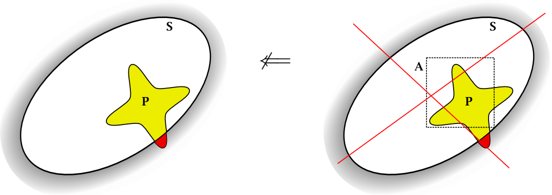
We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

Soundness and false alarms



$P \not\subseteq S$

$A \subseteq S$
false negative
cannot occur

Goal: prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

The analaysis is sound: no false negative reported!

Getting it right? eBPF example

eBPF:

- a virtual machine inside the Linux kernel
- can run arbitrary code in kernel mode
- very low-level, can perform arbitrary pointer arithmetic (flat memory model)
- run sandboxed to protect agains bugs and attacks

In theory:

- a static analysis checks bytecode safety before execution
- includes an interval analysis for pointers

Getting it *not* right! eBPF example

Bound computation for bit-shift >>:[7]

```
case BPF_RSH:
  if (min_val < 0 || dst_reg->min_value < 0)
    dst_reg->min_value = BPF_REGISTER_MIN_RANGE;
  else
    dst_reg->min_value = (u64)(dst_reg->min_value) >> min_val;
  if (dst_reg->max_value != BPF_REGISTER_MAX_RANGE)
    dst_reg->max_value >>= max_val;
  break;
```

Due to large amount of bugs in the static analysis,
a dynamic analysis has been added...
which exploits the (unsound) results from the static analysis...

Lesson

Use abstract interpretation to make analyses sound by construction!

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[7] https://www.zerodayinitiative.com/blog/2021/1/18/zdi-20-1440-an-incorrect-calculation-bug-in-the-linux-kernel-ebpf-verifier

Example tools

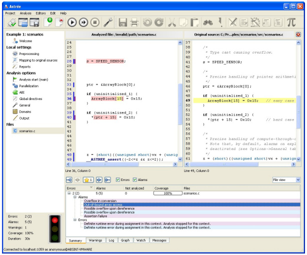
Astrée

Astrée: developed at ENS & INRIA by P. Cousot & al.

- analyzes embedded critical C software
  subset of C, no memory allocation, no recursivity → simpler semantics
- checks for run-time errors
  arithmetic overflows, array overflows, divisions by 0, pointer errors, etc. → non-functional
- specialized for control / command software
  with zero false alarm goal
  application domain specific abstractions

Airbus A380

2001–2004: academic success
proof of absence of RTE
on flight command



Analyzed file: /invalid/path/scenarios.c

```
    s = SPEED_SENSOR;




    ptr = &ArrayBlock[0];

    if (uninitialized_1) {
      ArrayBlock[15] = 0x15;
    }

    if (uninitialized_2) {
      *(ptr + 15) = 0x10;
    }






    z = (short)((unsigned short)vx + (unsig
    __ASTREE_assert((-2<=z && z<=2));
```

Line 36, Column 0

Original source: C:/Pr...ples/scenarios/src/scenarios.c

```
    /*
     * Type cast causing overflow.
     */
    s = SPEED_SENSOR;

    /*
     * Precise handling of pointer arithmeti
     */
    ptr = &ArrayBlock[0];

    if (uninitialized_1) {
      ArrayBlock[15] = 0x15;    // easy case
    }

    if (uninitialized_2) {
      *(ptr + 15) = 0x10;       // hard case
    }

    /*
     * Precise handling of compute-through-c
     * Note that, by default, alarms on expl
     * deactivated (see Options->General tab
     */
    z = (short)((unsigned short)vx + (unsign
```

Line 49, Column 0

☑ Errors ☑ Alarms

Errors	Alarms	Not analyzed	Coverage	Files
2 (2)	5 (5)	0	100%	scenarios.c

- Alarms
  - Overflow in conversion
  - Out-of-bound array access
  - Possible overflow upon dereference
  - Possible overflow upon dereference
  - Assertion failure
- Errors
  - Definite runtime error during assignment in this context. Analysis stopped for this context.
  - Definite runtime error during assignment in this context. Analysis stopped for this context.

Errors: 2 (2)
Alarms: 5 (5)
Warnings: 1
Coverage: 100%
Duration: 30s

Connected to localhost:1059 as anonymous@ABSINT-VMWARE

2009: industrialization

AbsInt

Infer.AI

Infer: `http://fbinfer.com/`

- developed at Facebook (team formerly at Monoidics)
- Infer.AI is an analysis framework based on abstract interpretation
- open-source since 2015
- analyzes Java, C, C++, and Objective-C
- checks ThreadSafety (Java), Initalisation Order (C++), etc.
- modular, bottom-up interprocedural analysis
- targets the analysis of merge requests (small bits at a time)
- favors speed over soundness

  pragmatic choices, based on "what engineers want"
  no requirements for certification, unlike the avionics industry. . .
- used in production

Frama-C

Frama-C: `https://frama-c.com/`

- developed at CEA
- open-source
- analyzes C
- combines abstract interpretation and deductive methods
- has a specification language (ACSL) for functional verification
- used in industrial applications

Example research project: MOPSA

Modular Open Platform For Static Analysis
developed at Sorbonne University: https://mopsa.lip6.fr/

An abstract interpreter prototype tool for research and education

- extendable to new properties and new languages
- help developing, reusing, combining abstractions
- open-source: https://gitlab.com/mopsa/mopsa-analyzer

Currently available: (not fully scalable!)

- C analysis for run-time error detection
- Python analysis (supports a large subset of Python 3, and a small subset of its library)
- analysis of programs mixing C and Python

On-going research: (not public yet, various level of maturity)

- patch and portability analysis for C
- analysis of smart-contracts (Michelson language for the Tezos blockchain)
- security-related properties

Course organisation

Teaching team

Caterina Urban

Jérôme Feret

Antoine Miné

Xavier Rival

Syllabus and exams

`https://www-apr.lip6.fr/~mine/enseignement/mpri/2022-2023`

Visit regularly for:

- latest information on course dates and modalities
- course material (slides)
- optional course assignments and reading
- internship proposals

Exams:

- 50%: written mid-term exam (3h)
- 50%: oral final exam
  (read a scientific article, present it, answer questions)

Course material

Available on the web page:

- main material: slides
- course notes

  cover mainly foundations and numeric abstract domains
  based on:

  A. Miné. *Tutorial on Static Inference of Numeric Invariants by Abstract Interpretation*. In Foundations and Trends in Programming Languages, 4(3–4), 120–372. Now Publishers.

- recommended reading on theory and applications:

  J. Bertrane, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, X. Rival. *Static analysis and verification of aerospace software by abstract interpretation.* In Foundations and Trends in Programming Languages, 2(2–3), 71–190, 2015. Now Publishers.

Course assignments (self-evaluation)

On the web page, **highly recommended** homework

- exercises: prove a theorem, solve a former exam, etc.
- reading assignments: an article related to the course
- experiments: use a tool

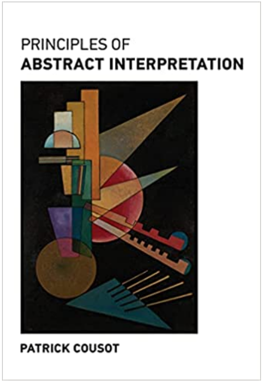
Also:

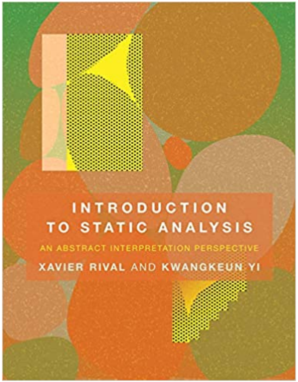
- previous exams, with correction
- example programming project
  (abstract interpreter for a toy language in OCaml)

Principle: self-evaluation

- no credit
- not corrected by the teachers

Books!



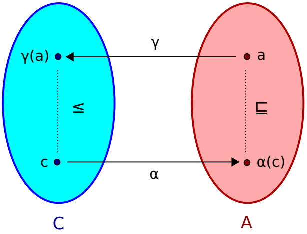


1. P. Cousot. Principles of Abstract Interpretation. 832 pages. The MIT Press. Sept. 2021.
2. X. Rival and K. Yi. Introduction to Static Analysis: An Abstract Interpretation Perspective. 320 pages. The MIT Press. Feb, 2020.

Course plan (1/8)

Foundations of abstract interpretation: (courses 1 & 2)

- mathematical background: order theory and fixpoints
- formalization of abstraction, soundness
- program semantics and program properties
- hierarchy of collecting semantics



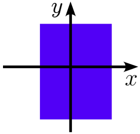
Course plan (2/8)

Bricks of abstraction: numerical domains

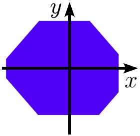
simple domains

relational domains

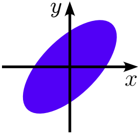
specific domains



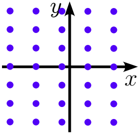
Intervals
$x \in [a, b]$



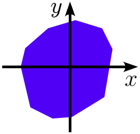
Octagons
$\pm x \pm y \leq c$



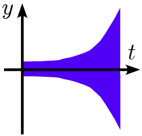
Ellipsoids
digital filters



Congruences
$x \in a\mathbb{Z} + b$



Polyhedra
$\sum_i \alpha_i x_i \leq \beta$

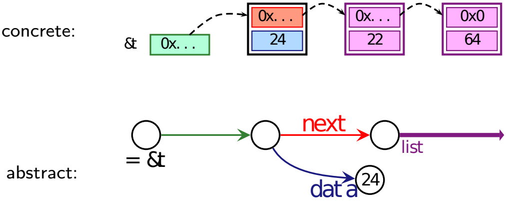


Exponentials
rounding errors

Course plan (3/8)

Bricks of abstraction: memory abstractions

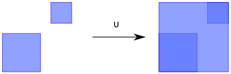
- beyond numeric: reason on arrays, lists, trees, graphs, . . .
- challenges: variety of structures, destructive updates
- logical tools:
    - separation logics (a logic tailored for describing memory)
    - parametric three valued logics (representing arbitrary graphs)
- abstract domains based on these logics



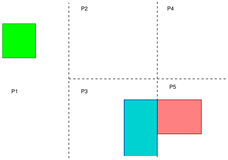
Course plan (4/8)

Bricks of abstraction: partitioning abstractions

- most abstract domains are not distributive
  $\Longrightarrow$ reasoning over disjunctions loses precision
- first solution: add disjunctions to any abstract domain
  $\Longrightarrow$ expressive but costly
- second solution: partitioning
  conjunctions of implications as logical predicates
  (partitioning may be based on many semantic criteria)



loss of precision

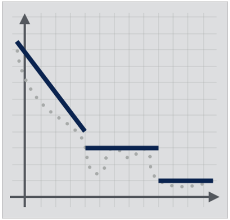


partitioning

Course plan (5/8)

Analyses: abstract interpretation for liveness properties

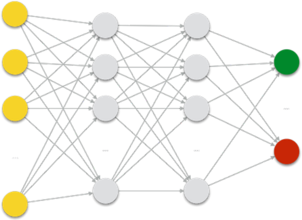
- beyond safety (e.g., absence of errors)
  we prove that programs (eventually) do something good
- abstract domains to reason about program termination
  inference of ranking functions



- generalization to other liveness properties
  (e.g., expressed in CTL)

Course plan (6/8)

Analyses: static analysis of neural networks

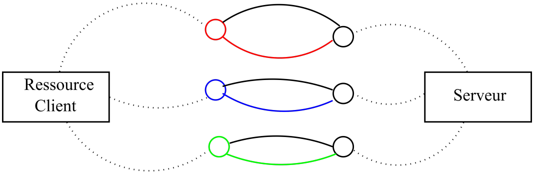


- verification of local robustness against adversarial examples
- fairness certification
  (special case of global robustness verification)
- verification of functional properties

Course plan (7/8)

Analyses: analysis of mobile systems

- dynamic creation of components and links
- analyze the links between components
    - distinguish between recursive components
    - abstractions as sets of words
- bound the number of components
  using numeric relations



Course plan (8/8)

Analyses: static analysis for security

- challenge: security properties are diverse
  from information leakage to unwanted execution of malicious code
  and more complex than safety and liveness
- the framework of hyperproperties can express security
- apply abstract interpretation to reason over non-interference

Internship proposals

Possibility of Master 2 internships at ENS or Sorbonne Université.

Example topics:

- Automatic inference of input data assumptions
- Fairness certification of machine-learned software
- Static analysis of medical data processing software
- Incremental static analysis
- Static analysis for multi-language programs
- ...

Formal proposals will be available on the course page
and discussed during the courses
also: discuss with your teachers for tailor-made subjects.