Introduction

MPRI 2–6: Abstract Interpretation, application to verification and static analysis

Antoine Miné

year 2015-2016

course 01 16 September 2015

Motivating program verification

The cost of software failure

- Patriot MIM-104 failure, 25 February 1991 (death of 28 soldiers¹)
- Ariane 5 failure, 4 June 1996 (cost estimated at more than 370 000 000 US\$²)
- Toyota electronic throttle control system failure, 2005 (at least 89 death³)
- Heartbleed bug in OpenSSL, April 2014
- Stagefright bug in Android, Summer 2015 (multiple array overflows in 900 million devices, some exploitable)
- economic cost of software bugs is tremendous⁴

¹R. Skeel. "Roundoff Error and the Patriot Missile". SIAM News, volume 25, nr 4.

- ²M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.
- ³CBSNews. Toyota "Unintended Acceleration" Has Killed 89. 20 March 2014.

⁴NIST. Software errors cost U.S. economy \$59.5 billion annually. Tech. report, NIST Planning Report, 2002.

Introduction

Zoom on: Ariane 5, Flight 501



Maiden flight of the Ariane 5 Launcher, 4 June 1996.

Introduction

Zoom on: Ariane 5, Flight 501



40s after launch...

Introduction

Zoom on: Ariane 5, Flight 501

Cause: software error⁵

 arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types⁶

```
P_M_DERIVE(T_ALG.E_BH) :=
UC_16S_EN_16NS (TDB.T_ENTIER_16S
  ((1.0/C_M_LSB_BH) * G_M_INF0_DERIVE(T_ALG.E_BH)));
```

- software exception not caught
 - \implies computer switched off
- all backup computers run the same software
 - \implies all computers switched off, no guidance
 - \implies rocket self-destructs

⁵J.-L. Lions et al., Ariane 501 Inquiry Board report.

⁶J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d'informatique de l'ENS, 2010.

How can we avoid such failures?

• Choose a safe programming language.

C (low level) / Ada, Java (high level)

• Carefully design the software.

many software development methods exist

• Test the software extensively.

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• Choose a safe programming language.

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many software development methods exist yet, critical embedded software follow strict development processes

• Test the software extensively.

yet, the erroneous code was well tested... on Ariane 4

\implies not sufficient!

How can we avoid such failures?

• Choose a safe programming language.

C (low level) / Ada, Java (high level) yet, Ariane 5 software is written in Ada

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many software development methods exist yet, critical embedded software follow strict development processes

• Test the software extensively.

yet, the erroneous code was well tested... on Ariane 4

\implies not sufficient!

We should use formal methods.

provide rigorous, mathematical insurance

Proving program properties

assume X in [0,1000]; I := 0; while I < X do I := I + 2;

assert I in [0,?]

Goal: find a bound property, sufficient to express the absence of overflow

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⁷ R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

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```
assume X in [0,1000];

\{X \in [0,1000]\}

I := 0;

\{X \in [0,1000], I = 0\}

while I < X do

\{X \in [0,1000], I \in [0,998]\}

I := I + 2;

\{X \in [0,1000], I \in [2,1000]\}

\{X \in [0,1000], I \in [0,1000]\}

assert I in [0,1000]
```



Robert Floyd⁷

invariant: property true of all the executions of the program

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\{X \in [0,1000], I \in [0,1000]\}

assert I in [0,1000]
```



Robert Floyd⁷

invariant: property true of all the executions of the program **issue**: if I = 997 at a loop iteration, I = 999 at the next

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```
assume X in [0,1000];

\{X \in [0,1000]\}

I := 0;

\{X \in [0,1000], I = 0\}

while I < X do

\{X \in [0,1000], I \in \{0,2,\ldots,996,998\}\}

I := I + 2;

\{X \in [0,1000], I \in \{2,4,\ldots,998,1000\}\}

\{X \in [0,1000], I \in \{0,2,\ldots,998,1000\}\}

assert I in [0,1000]
```



Robert Floyd⁷

inductive invariant: invariant that can be proved to hold by induction on loop iterates

(if $I \in S$ at a loop iteration, then $I \in S$ at the next loop iteration)

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⁷ R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

Logics and programs





Tony Hoare⁸

- sound logic to prove program properties, (rel.) complete
- proofs can be partially automated (at least proof checking) (e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)

⁸C. A. R. Hoare. "An Axiomatic Basis for Computer Programming". Commun. ACM 12(10): 576–580 (1969).

Logics and programs





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- sound logic to prove program properties, (rel.) complete
- proofs can be partially automated (at least proof checking) (e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)
- requires annotations and interaction with a prover even manual annotation is not practical for large programs

⁸C. A. R. Hoare. "An Axiomatic Basis for Computer Programming". Commun. ACM 12(10): 576–580 (1969).

A calculs of program properties

 $wlp(\mathbf{X} := \mathbf{e}, P) \stackrel{\text{def}}{=} P[\mathbf{e}/\mathbf{X}]$ $wlp(\mathbf{C}_1; \mathbf{C}_2, P) \stackrel{\text{def}}{=} wlp(\mathbf{C}_1, wlp(\mathbf{C}_2, P))$ $wlp(\text{while } \mathbf{e} \text{ do } \mathbf{C}, P) \stackrel{\text{def}}{=}$ $I \land ((\mathbf{e} \land I) \implies wlp(\mathbf{C}, I)) \land ((\neg \mathbf{e} \land I) \implies P)$



Edsger W. Dijkstra⁹

• predicate transformer semantics

propagate predicates on states through the program

• weakest (liberal) precondition

backwards, from property to prove to condition for program correctness

calculs that can be mostly automated

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⁹E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

A calculs of program properties

$$wlp(X := e, P) \stackrel{\text{def}}{=} P[e/X]$$

$$wlp(C_1; C_2, P) \stackrel{\text{def}}{=} wlp(C_1, wlp(C_2, P))$$

$$wlp(while e do C, P) \stackrel{\text{def}}{=}$$

$$I \land ((e \land I) \implies wlp(C, I)) \land ((\neg e \land I) \implies P)$$



Edsger W. Dijkstra⁹

• predicate transformer semantics

propagate predicates on states through the program

• weakest (liberal) precondition

backwards, from property to prove to condition for program correctness

- calculs that can be mostly automated, except for:
 - user annotations for inductive loop invariants
 - function annotations (modular inference)
- academic success: complex (functional) but local properties
- industry success: simple and local properties

⁹E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

Limit to automation

Computers, programs, data

 $O(P, D) \in \{yes, no, \bot\}$



The computer *O* runs the program *P* on the data *D* and answers (yes, no)... or does not answer (\perp) .

Computers, programs, data

$O(P, D) \in \{yes, no, \bot\}$



Note that programs are also a kind of data! They can be fed to other programs. (e.g., to compilers) Static analyzer *A*.

Given a program *P*:

- $O(A, P) = yes \iff \forall D, O(P, D)$ is safe
- $O(A, P) \neq \bot$ (the static analysis always terminates)

Static analyzer A.

Given a program *P*:

- $O(A, P) = yes \iff \forall D, O(P, D)$ is safe
- $O(A, P) \neq \bot$ (the static analysis always terminates)

 \implies *P* is proved safe even before it is run!



There cannot exist a static analyzer A proving the termination of every terminating program P.



Alan Turing¹⁰

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Introduction

¹⁰ A. M. Turing. "Computability and definability". The Journal of Symbolic Logic, vol. 2, pp. 153–163, (1937).

¹¹H. G. Rice. "Classes of Recursively Enumerable Sets and Their Decision Problems." Trans. Amer. Math. Soc. 74, 358-366, 1953.

There cannot exist a static analyzer A proving the termination of every terminating program P.

$$\frac{\text{Proof sketch:}}{A(P \cdot D) : O(A, P \cdot D)} = \begin{vmatrix} \text{yes if } O(P, D) \neq \bot \\ \text{no otherwise} \end{vmatrix}$$

A'(X) : while $A(X \cdot X)$ do nothing; no

do we have $O(A', A') = \bot$ or $\neq \bot$? neither! $\implies A$ cannot exist



Alan Turing¹⁰

All "interesting" properties are undecidable!¹¹

Introduction

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¹¹H. G. Rice, "Classes of Recursively Enumerable Sets and Their Decision Problems," Trans. Amer. Math. Soc. 74, 358-366, 1953.

Approximation

An approximate static analyzer A always answers in finite time $(\neq \bot)$:

• either yes: the program *P* is definitely safe

(soundness)

• either *no*: I don't know

(incompleteness)

Sufficient to prove the safety of (some) programs. Fails on infinitely many programs... An approximate static analyzer A always answers in finite time $(\neq \bot)$:

• either yes: the program *P* is definitely safe

(soundness)

• either *no*: I don't know

(incompleteness)

Sufficient to prove the safety of (some) programs. Fails on infinitely many programs...

- \implies We should adapt the analyzer A to
 - a class of programs to verify, and
 - a class of safety properties to check.



Patrick Cousot¹²

présentée à	
Université Scientifique et Médicale de Grenoble	
Institut National Polytechnique de Grenoble	
pour abtenir in grude de accrean as acanas warigeneroues	
par	
Patrick COUSOT	
640	
METHODES ITERATIVES DE CONSTRUCTION	
ET D'APPROXIMATION DE POINTS FIXES	
D'OPERATEURS MONOTONES SUR UN TREILLIS,	
ANALYSE SEMANTIQUE DES PROGRAMMES.	
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President I L BOLLIET	
Examination : G. EDISLANDS	

B. LOWINO C. PAUR F. ROBERT

General theory of the approximation and comparison of program semantics:

- unifies many existing semantics
- allows the definition of new static analyses that are correct by construction

¹²P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

 (\mathcal{S}_0)

assume X in [0,1000];

$$S_i \in D = \mathcal{P}(\{I,X\} \to \mathbb{Z})$$
 (S_1)
 $S_0 = \{(i,x) | i, x \in \mathbb{Z}\}$
 $= \top$

 I := 0;
 $S_1 = \{(i,x) \in S_0 | x \in [0,1000]\}$
 $= F_1(S_0)$
 (S_2)
 $S_2 = \{(0,x) | \exists i, (i,x) \in S_1\}$
 $= F_2(S_1)$

 while (S_3) I < X do
 $S_3 = S_2 \cup S_5$
 $S_4 = \{(i,x) \in S_3 | i < x\}$
 $= F_4(S_3)$

 I := I + 2;
 $S_5 = \{(i+2,x) | (i,x) \in S_4\}$
 $= F_5(S_4)$
 (S_5)
 $S_6 = \{(i,x) \in S_3 | i \ge x\}$
 $= F_6(S_3)$

 program
 semantics

Concrete semantics $S_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I}, \mathtt{X}\} \to \mathbb{Z})$:

- strongest invariant (and an inductive invariant)
- not computable in general
- smallest solution of a system of equations



Abstract semantics $\mathcal{S}_{i}^{\sharp} \in \mathcal{D}^{\sharp}$:

- \mathcal{D}^{\sharp} is a subset of properties of interest (approximation) with a machine representation
- *F*[#]: D[#] → D[#] over-approximates the effect of *F*: D → D in D[#] (with effective algorithms)









concrete sets \mathcal{D} :

 $\{(0,3), (5.5,0), (12,7), \ldots\}$ $\begin{array}{ll} \text{abstract polyhedra } \mathcal{D}_{\rho}^{\sharp} \colon & 6X + 11Y \geq 33 \wedge \cdots \\ \text{abstract octagons } \mathcal{D}_{\sigma}^{\sharp} \colon & X + Y \geq 3 \wedge Y \geq 0 \wedge \cdots \end{array}$ abstract intervals \mathcal{D}_i^{\sharp} : $X \in [0, 12] \land Y \in [0, 8]$



concrete sets \mathcal{D} :

 $\{(0,3), (5.5,0), (12,7), \ldots\}$ abstract polyhedra \mathcal{D}_{p}^{\sharp} : $6X + 11Y \ge 33 \land \cdots$ exponential cost abstract octagons \mathcal{D}_{o}^{\sharp} : $X + Y \geq 3 \land Y \geq 0 \land \cdots$ cubic cost abstract intervals \mathcal{D}_i^{\sharp} : $X \in [0, 12] \land Y \in [0, 8]$

not computable linear cost

Trade-off between cost and expressiveness / precision

Correctness proof and false alarms



The program is correct (blue \cap red = \emptyset).

Correctness proof and false alarms



The program is correct (blue \cap red = \emptyset). The polyhedra domain can prove the correctness (cyan \cap red = \emptyset).

Correctness proof and false alarms



The program is correct (blue \cap red = \emptyset). The polyhedra domain can prove the correctness (cyan \cap red = \emptyset). The interval domain cannot (green \cap red $\neq \emptyset$, false alarm).

Introduction

Numeric abstract domain examples (cont.)



abstract semantics F^{\sharp} in the interval domain \mathcal{D}_{i}^{\sharp}

• $I \in D_i^{\sharp}$ is a pair of bounds $(\ell, h) \in \mathbb{Z}^2$ (for each variable) representing an interval $[\ell, h] \subseteq \mathbb{Z}$

• I:=I+2:
$$(\ell, h) \mapsto (\ell+2, h+2)$$

•
$$\cup^{\sharp}$$
: $(\ell_1, h_1) \cup^{\sharp} (\ell_2, h_2) = (\min(\ell_1, \ell_2), \max(h_1, h_2))$

• . .

Resolution by iteration and extrapolation

Challenge: the equation system is recursive: $\vec{S}^{\sharp} = \vec{F}^{\sharp}(\vec{S}^{\sharp})$. Solution: resolution by iteration: $\vec{S}^{\sharp 0} = \emptyset^{\sharp}, \vec{S}^{\sharp i+1} = \vec{F}^{\sharp}(\vec{S}^{\sharp i})$. e.g., S_3^{\sharp} : $I \in \emptyset$, I = 0, $I \in [0, 2]$, $I \in [0, 4]$, ..., $I \in [0, 1000]$

Resolution by iteration and extrapolation

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Challenge: infinite or very long sequence of iterates in \mathcal{D}^{\sharp}

Solution: extrapolation operator ∇

e.g., $[0,2] \bigtriangledown [0,4] = [0,+\infty[$

- remove unstable bounds and constraints
- ensures the convergence in finite time
- inductive reasoning (through generalisation)

Resolution by iteration and extrapolation

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 \implies effective solving method \longrightarrow static analyzer!

Other uses of abstract interpretation

- Analysis of dynamic memory data-structures (shape analysis).
- Analysis of parallel, distributed, and multi-thread programs.
- Analysis of probabilistic programs.
- Analysis of biological systems.
- Security analysis (information flow).
- Termination analysis.
- Cost analysis.
- Analyses to enable compiler optimisations.

• . . .

Some static analysis tools based on Abstract Interpretation

The Astrée static analyzer

🔺 Astrée					
Project Analysis Editors Edit He	b				
ACC POOPHEDCXJC*! Xexee T					
Example 1: scenarios	Analyzed file: /invalid/path/scenarios.c	Original source: C:/Prples/scenarios/src/scenarios.c 🧕			
So Welcome	24	37			
Local settings	25	38 /*			
Preprocessing	26	39 * Type cast causing overflow.			
2 Mapping to original sources	28 s = SPEED SENSOR;	40 */ 41 * = SPRED SENSOR:			
Z, Reports	29	42			
Analysis options	30	43 /*			
Analysis start (main)	32	44 * Precise handling of pointer arithmeti			
Parallelization	<pre>33 ptr = &ArrayBlock[0];</pre>	46 ptr = &ArrayBlock[0];			
abi 🖉	34 25 if (uninitialized t) (47			
Z Global directives	36 ArrayBlock[15] = 0x15;	48 if (uninitialized_1) (
J General	37)	50)			
E Dornains	38	51			
/ Output	40 *(ptr + 15) = 0x10;	52 if (uninitialized_2) (
Files	41)	53 *(ptr + 15) = 0x10; // naru case			
	42	55			
Scenarios.c	43	56 /*			
	45	57 * Precise handling of compute-through-c			
	46	50 " Note that, by derault, diarms on expl 59 * deactivated (see Ontions->General tak			
	48 z = (short) ((unsigned short) vx + (unsig	60 */			
	49 ASTREE_assert((-2<=z 66 z<=2));	61 z = (short)((unsigned short)vx + (unsign			
	<	K			
	Line 36, Column 0	Line 49, Column 0			
	┥ 🗣 🏫 1 🕪 🔛 🗹 Errors 🗸 Alarms	File view 💌			
	Errors Alarms Not analyzed Coverage	Files			
	E 2 (2) 5 (5) 0 100%] scenarios.c			
	Overflow in conversion				
	Out-ot-bound array access Possible overflow upon dereference				
Errorr: 2(2)	Possible overflow upon dereference Assertion failure				
Alarms: 5(5)	Errors				
Warnings: 1	 Definite runtime error during assignment in this context. Analysis stop Definite runtime error during assignment in this context. Analysis stop 	ped for this context. ped for this context.			
Coverage: 100%					
Duration: 30s	Summary Warnings Log Graph Watch Messages				
Connected to localhost:1059 as anonymou	#DABSINT-YMWARE				

Introduction

Analyseur statique de programmes temps-réels embarqués

(static analyzer for real-time embedded software)

- developed at ENS
 B. Blanchet, P. Cousot, R. Cousot, J. Feret,
 L. Mauborgne, D. Monniaux, A. Miné, X. Rival
- industrialized and made commercially available by AbsInt





The Astrée static analyzer

Specialized:

• for the analysis of run-time errors

(arithmetic overflows, array overflows, divisions by 0, etc.)

• on embedded critical C software

(no dynamic memory allocation, no recursivity)

• in particular on control / command software

(reactive programs, intensive floating-point computations)

• intended for validation

(analysis does not miss any error and tries to minimise false alarms)

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Approximately 40 abstract domains are used at the same time:

- numeric domains (intervals, octagons, ellipsoids, etc.)
- boolean domains
- domains expressing properties on the history of computations

Astrée applications



Airbus A340-300 (2003)



Airbus A380 (2004)

- size: from 70 000 to 860 000 lines of C
- analysis time: from 45mn to \simeq 40h
- 0 alarm: proof of absence of run-time error

Fluctuat

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Static analysis of the accuracy of floating-point computations:

- bound the range of variables
- bound the rounding errors wrt. real computation
- track the origin of rounding errors (which operation contributes to most error, target for improvements)
- uses specific abstract domains

(affine arithmetic, zonotopes)

- developed at CEA-LIST (E. Goubault, S. Putot)
- industrial use (Airbus)

Clousot: CodeContract static checker



Clousot: CodeContract static checker

CodeContracts:

• assertion language for .NET (C#, VB, etc.)

(pre-conditions, post-conditions, invariants)

dynamic checking

(insert run-time checks)

static checking

(modular abstract interpretation)

automatic inference

(abstract interpretation to infer necessary preconditions backwards)

- developed at Microsoft Research (M. Fahndrich, F. Logozzo)
- part of .NET Framework 4.0
- integrated to Visual Studio