Introduction

MPRI 2–6: Abstract Interpretation, application to verification and static analysis

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course 01 16 September 2015

Motivating program verification

The cost of software failure

- Patriot MIM-104 failure, 25 February 1991 $(d$ eath of 28 soldiers¹)
- Ariane 5 failure, 4 June 1996 (cost estimated at more than 370 000 000 $US2)
- Toyota electronic throttle control system failure, 2005 $(at least 89 death³)$
- Heartbleed bug in OpenSSL, April 2014
- Stagefright bug in Android, Summer 2015 (multiple array overflows in 900 million devices, some exploitable)
- **e** economic cost of software bugs is tremendous⁴

 $^{\rm 1}$ R. Skeel. "Roundoff Error and the Patriot Missile". SIAM News, volume 25, nr 4.

- ²M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.
- 3 CBSNews. Toyota "Unintended Acceleration" Has Killed 89. 20 March 2014.

4 NIST. Software errors cost U.S. economy \$59.5 billion annually. Tech. report, NIST Planning Report, 2002.

course 01 ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$ ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$ ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$

Zoom on: Ariane 5, Flight 501

Maiden flight of the Ariane 5 Launcher, 4 June 1996.

course 01 ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$ ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$ ${\color{red} \begin{array}{c} \text{Introduction} \end{array}}$

Zoom on: Ariane 5, Flight 501

40s after launch. . .

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Cause: software error⁵

• arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types 6

```
P_M_DERIVE(T_ALG.E_BH) :=
  UC 16S EN 16NS (TDB.T ENTIER 16S
    ((1.0/CMLSB_BH) * GMLINFO_DERIVE(T_ALG.E_BH)));
```
- software exception not caught
	- \implies computer switched off
- all backup computers run the same software
	- \implies all computers switched off, no guidance
	- =⇒ rocket self-destructs

⁵ J.-L. Lions et al., Ariane 501 Inquiry Board report.

e
6 J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d'informatique de l'ENS, 2010.

How can we avoid such failures?

• Choose a safe programming language.

C (low level) / Ada, Java (high level)

• Carefully design the software.

many software development methods exist

• Test the software extensively.

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yet, the erroneous code was well tested. . . on Ariane 4

\implies not sufficient!

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We should use **formal methods**.

provide rigorous, mathematical insurance

Proving program properties

assume X in [0,1000]; $I := 0$: while I < X do $I := I + 2;$

assert I in [0,?]

Goal: find a bound property, sufficient to express the absence of overflow

⁷ R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

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assume X in [0,1000];
\{X \in [0, 1000]\}I := 0:
\{X \in [0, 1000], I = 0\}while I < X do
    \{X \in [0, 1000], I \in [0, 998]\}I := I + 2;
    \{X \in [0, 1000], I \in [2, 1000]\}\{X \in [0, 1000], I \in [0, 1000]\}assert I in [0,1000]
```


Robert Floyd⁷

invariant: property true of all the executions of the program

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invariant: property true of all the executions of the program **issue**: if $I = 997$ at a loop iteration, $I = 999$ at the next

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\{X \in [0, 1000]\}I := 0:
\{X \in [0, 1000], I = 0\}while I < X do
     \{X \in [0, 1000], I \in \{0, 2, \ldots, 996, 998\}\}\I := I + 2;
     \{X \in [0, 1000], I \in \{2, 4, \ldots, 998, 1000\}\}\\{X \in [0, 1000], I \in \{0, 2, \ldots, 998, 1000\}\}\assert I in [0,1000]
```


Robert Floyd⁷

inductive invariant: invariant that can be proved to hold by induction on loop iterates

(if $I \in S$ at a loop iteration, then $I \in S$ at the next loop iteration)

⁷ R. W. Floyd. "Assigning meanings to programs". In Proc. Amer. Math. Soc. Symposia in Applied Mathematics, vol. 19, pp. 19–31, 1967.

Logics and programs

Tony Hoare⁸

- sound logic to prove program properties, (rel.) complete
- **proofs can be partially automated** (at least proof checking) (e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)

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⁸ C. A. R. Hoare. "An Axiomatic Basis for Computer Programming". Commun. ACM 12(10): 576–580 (1969).

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- **proofs can be partially automated** (at least proof checking) (e.g., using proof assistants: Coq, PVS, Isabelle, HOL, etc.)
- **•** requires annotations and interaction with a prover even manual annotation is not practical for large programs

⁸ C. A. R. Hoare. "An Axiomatic Basis for Computer Programming". Commun. ACM 12(10): 576–580 (1969).

A calculs of program properties

$$
wlp(\mathbf{X} := \mathbf{e}, P) \stackrel{\text{def}}{=} P[e/X]
$$

\n
$$
wlp(\mathbf{C}_1; \mathbf{C}_2, P) \stackrel{\text{def}}{=} wlp(\mathbf{C}_1, wlp(\mathbf{C}_2, P))
$$

\n
$$
wlp(\text{while } \mathbf{e} \text{ do } \mathbf{C}, P) \stackrel{\text{def}}{=} I \wedge ((e \wedge I) \implies wlp(\mathbf{C}, I)) \wedge ((\neg e \wedge I) \implies P)
$$

Edsger W. Dijkstra⁹

predicate transformer semantics

propagate predicates on states through the program

weakest (liberal) precondition

backwards, from property to prove to condition for program correctness

• calculs that can be mostly automated

⁹ E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

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• predicate transformer semantics

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- calculs that can be mostly automated, except for:
	- user annotations for inductive loop invariants
	- **function annotations** (modular inference)
- academic success: complex (functional) but local properties
- industry success: simple and local properties

9 E. W. Dijkstra. "Guarded commands, nondeterminacy and formal derivation of programs". EWD472. Commun. ACM 18(8): 453-457 (1975).

Limit to automation

Computers, programs, data

$O(P, D) \in \{yes, no, \perp\}$

The computer O runs the program P on the data D and answers (yes, no) ... or does not answer (\perp) .

Computers, programs, data

$O(P, D) \in \{yes, no, \perp\}$

Note that programs are also a kind of data! They can be fed to other programs. (e.g., to compilers) Static analyzer A.

Given a program P:

- \odot $O(A, P) = yes \iff \forall D, O(P, D)$ is safe
- \bullet $O(A, P) \neq \perp$ (the static analysis always terminates)

Static analyzer A.

Given a program P:

- \odot $O(A, P) = yes \iff \forall D, O(P, D)$ is safe
- \bullet $O(A, P) \neq \perp$ (the static analysis always terminates)

 $\implies P$ is proved safe even before it is run!

There cannot exist a static analyzer A proving the termination of every terminating program P.

Alan Turing¹⁰

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^{10&}lt;br>A. M. Turing. "Computability and definability". The Journal of Symbolic Logic, vol. 2, pp. 153–163, (1937).

¹¹H. G. Rice. "Classes of Recursively Enumerable Sets and Their Decision Problems." Trans. Amer. Math. Soc. 74, 358-366, 1953.

There cannot exist a static analyzer A proving the termination of every terminating program P.

Proof sketch:

$$
A(P \cdot D) : O(A, P \cdot D) = \begin{vmatrix} yes & \text{if } O(P, D) \neq \bot \\ no & \text{otherwise} \end{vmatrix}
$$

 $A'(X)$: while $A(X \cdot X)$ do nothing; no

do we have $O(A',A') = \bot$ or $\neq \bot ?$ neither! \implies A cannot exist Alan Turing ¹⁰

All "interesting" properties are undecidable!¹¹

^{10&}lt;br>A. M. Turing. "Computability and definability". The Journal of Symbolic Logic, vol. 2, pp. 153-163, (1937).

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Approximation

An approximate static analyzer \overline{A} always answers in finite time $(\neq \bot)$:

- either y es: the program P is definitely safe (soundness)
	-
- either *no*: I don't know

Sufficient to prove the safety of (some) programs. Fails on infinitely many programs. . .

An approximate static analyzer \overline{A} always answers in finite time $(\neq \perp)$:

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Sufficient to prove the safety of (some) programs. Fails on infinitely many programs. . .

- \implies We should adapt the analyzer A to
	- a class of programs to verify, and
	- a class of safety properties to check.

General theory of the approximation and comparison of program semantics:

- **•** unifies many existing semantics
- allows the definition of new static analyses that are correct by construction

¹²P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

```
(S_0)assume X in [0,1000];
 (S_1)I := 0;(S_2)while (\mathcal{S}_3) I < X do
      (S_4)I := I + 2;(\mathcal{S}_5)(S_6)program
```
 (S_0)

assume X in [0,1000];	$S_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \to \mathbb{Z})$	$= \top$	
$[S_1]$	$S_0 = \{(i, x) i, x \in \mathbb{Z}\}$	$= \top$	
$[S_2]$	$S_2 = \{(0, x) \exists i, (i, x) \in S_1\}$	$= F_1(S_0)$	
$[S_2]$	$S_2 = \{(0, x) \exists i, (i, x) \in S_1\}$	$= F_2(S_1)$	
while $[S_3]$ I < X do	$S_3 = S_2 \cup S_5$	$S_4 = \{(i, x) \in S_3 i < x\}$	$= F_4(S_3)$
$[S_5]$	$S_6 = \{(i + 2, x) (i, x) \in S_4\}$	$= F_5(S_4)$	
$[S_6]$	$S_6 = \{(i, x) \in S_3 i \geq x\}$	$= F_6(S_3)$	
$[S_6]$	$S_6 = \{(i, x) \in S_3 i \geq x\}$	$= F_6(S_3)$	

Concrete semantics $S_i \in \mathcal{D} = \mathcal{P}(\lbrace \mathtt{I}, \mathtt{X} \rbrace \rightarrow \mathbb{Z})$:

- **strongest invariant** (and an inductive invariant)
- o not computable in general
- smallest solution of a system of equations

 (S_0) assume X in [0,1000]; (S_1) $I := 0;$ (S_2) while (\mathcal{S}_3) I < X do (S_4) $I := I + 2;$ (\mathcal{S}_5) (S_6) $\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$ $\mathcal{S}_0^\sharp = \top_1^\sharp$ $\mathcal{S}_{1}^{\sharp}=\mathcal{F}_{1}^{\sharp}(\mathcal{S}_{0}^{\sharp})$ $\mathcal{S}_{2}^{\sharp}=\mathcal{F}_{2}^{\sharp}(\mathcal{S}_{1}^{\sharp})$ $\mathcal{S}_{3}^{\sharp} = \mathcal{S}_{2}^{\sharp} \cup^{\sharp} \mathcal{S}_{5}^{\sharp}$ $\mathcal{S}_4^{\sharp} = \mathcal{F}_4^{\sharp}(\mathcal{S}_3^{\sharp})$ $\mathcal{S}_{5}^{\sharp}=\mathcal{F}_{5}^{\sharp}(\mathcal{S}_{4}^{\sharp})$ $\mathcal{S}_6^\sharp = \mathcal{F}_6^\sharp(\mathcal{S}_3^\sharp)$ program semantics

Abstract semantics $\mathcal{S}_{i}^{\sharp} \in \mathcal{D}^{\sharp}$:

- \mathcal{D}^{\sharp} is a subset of properties of interest $\qquad \qquad \text{(approximation)}$ with a machine representation
- $\mathcal{F}^\sharp:\mathcal{D}^\sharp\to\mathcal{D}^\sharp$ over-approximates the effect of $\mathcal{F}:\mathcal{D}\to\mathcal{D}$ in \mathcal{D}^\sharp (with effective algorithms)

concrete sets \mathcal{D} : $\{(0, 3), (5.5, 0), (12, 7), \ldots\}$ abstract polyhedra $\mathcal{D}_\rho^\sharp \colon \quad 6X + 11Y \geq 33 \wedge \cdots$ abstract octagons $\mathcal{D}_{\circ}^{\sharp}: \quad X + Y \geq 3 \wedge Y \geq 0 \wedge \cdots$ abstract intervals $\mathcal{D}_i^{\sharp}: \quad X \in [0,12] \wedge Y \in [0,8]$

concrete sets $D: \{ (0, 3), (5.5, 0), (12, 7), ...\}$ not computable abstract polyhedra $\mathcal{D}_\rho^\sharp \colon \quad 6X + 11Y \geq 33 \wedge \cdots \qquad \qquad$ exponential cost abstract octagons $\mathcal{D}_o^{\sharp}: \quad X + Y \geq 3 \wedge Y \geq 0 \wedge \cdots$ cubic cost abstract intervals \mathcal{D}_i^{\sharp} : $\qquad \mathcal{X} \in [0,12] \wedge \mathcal{Y} \in [0,8]$ linear cost

Trade-off between cost and expressiveness / precision

Correctness proof and false alarms

The program is correct (blue \cap red = \emptyset).

Correctness proof and false alarms

The program is correct (blue \cap red = \emptyset). The polyhedra domain can prove the correctness (cyan \cap red = \emptyset).

Correctness proof and false alarms

The program is correct (blue \cap red = \emptyset). The polyhedra domain can prove the correctness (cyan \cap red = \emptyset). The interval domain cannot (green \cap red $\neq \emptyset$, false alarm).

Numeric abstract domain examples (cont.)

abstract semantics \mathcal{F}^\sharp in the interval domain \mathcal{D}_i^\sharp i

 $I \in \mathcal{D}_i^{\sharp}$ **is a pair of bounds** $(\ell, h) \in \mathbb{Z}^2$ (for each variable) representing an interval $[\ell, h] \subseteq \mathbb{Z}$

$$
\bullet \quad \mathbf{I} := \mathbf{I} + 2: \ (\ell, h) \mapsto (\ell + 2, h + 2)
$$

•
$$
\bigcup^{\sharp}
$$
: $(\ell_1, h_1) \bigcup^{\sharp} (\ell_2, h_2) = (\min(\ell_1, \ell_2), \max(h_1, h_2))$

٠ . . .

Resolution by iteration and extrapolation

Challenge: the equation system is recursive: $\vec{S}^{\sharp} = \vec{F}^{\sharp}(\vec{S}^{\sharp}).$ $\underline{\mathsf{Solution:}}$ resolution by iteration: $\vec{\mathcal{S}}^{\sharp\,0}=\emptyset^{\sharp},$ $\vec{\mathcal{S}}^{\sharp\,i+1}=\vec{\mathit{F}}^{\sharp}(\vec{\mathcal{S}}^{\sharp\,i}).$ e.g., \mathcal{S}_{3}^{\sharp} $\frac{1}{3}$: $I \in \emptyset$, $I = 0$, $I \in [0, 2]$, $I \in [0, 4]$, ..., $I \in [0, 1000]$

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Challenge: infinite or very long sequence of iterates in \mathcal{D}^{\sharp}

Solution: extrapolation operator ∇

e.g., $[0, 2] \triangledown [0, 4] = [0, +\infty]$

- **•** remove unstable bounds and constraints
- **e** ensures the convergence in finite time
- inductive reasoning (through generalisation)

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 \implies effective solving method \longrightarrow static analyzer!

Other uses of abstract interpretation

- Analysis of dynamic memory data-structures (shape analysis).
- Analysis of parallel, distributed, and multi-thread programs.
- Analysis of probabilistic programs.
- Analysis of biological systems.
- Security analysis (*information flow*).
- **•** Termination analysis.
- **•** Cost analysis.
- Analyses to enable compiler optimisations.

 \bullet . . .

Some static analysis tools based on Abstract Interpretation

The Astrée static analyzer

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Analyseur statique de programmes temps-réels embarqués

(static analyzer for real-time embedded software)

o developed at ENS B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, D. Monniaux, A. Miné, X. Rival

• industrialized and made commercially available by AbsInt

www.astree.ens.fr

The Astrée static analyzer

Specialized:

• for the analysis of run-time errors

(arithmetic overflows, array overflows, divisions by 0, etc.)

on embedded critical C software

(no dynamic memory allocation, no recursivity)

• in particular on control / command software

(reactive programs, intensive floating-point computations)

• intended for validation

(analysis does not miss any error and tries to minimise false alarms)

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Approximately 40 abstract domains are used at the same time:

- **•** numeric domains (intervals, octagons, ellipsoids, etc.)
- **•** boolean domains
- domains expressing properties on the history of computations

Astrée applications

Airbus A340-300 (2003) Airbus A380 (2004)

- size: from 70,000 to 860,000 lines of C
- analysis time: from 45mn to \simeq 40h
- 0 alarm: proof of absence of run-time error

Fluctuat

Static analysis of the **accuracy of floating-point computations**:

- bound the range of variables
- bound the rounding errors wrt. real computation
- track the origin of rounding errors (which operation contributes to most error, target for improvements)
- uses specific abstract domains

(affine arithmetic, zonotopes)

- **o** developed at CEA-LIST (E. Goubault, S. Putot)
- industrial use (Airbus)

Clousot: CodeContract static checker

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CodeContracts:

• assertion language for .NET $(C#, VB, etc.)$

(pre-conditions, post-conditions, invariants)

o dynamic checking

(insert run-time checks)

• static checking

(modular abstract interpretation)

automatic inference

(abstract interpretation to infer necessary preconditions backwards)

- developed at Microsoft Research (M. Fahndrich, F. Logozzo)
- part of .NET Framework 4.0
- integrated to Visual Studio