Exercices
MPRI 2-6

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Let Σ be a set of states. Given a transition relation $\tau \subseteq \Sigma \times \Sigma$, we denote by $\mathcal{T}[\tau]$ the set of partial finite traces obeying τ:

$$\mathcal{T}[\tau] \stackrel{\text{def}}{=} \{ (\sigma_0, \ldots, \sigma_n) \in \Sigma^+ \mid \forall i < n : (\sigma_i, \sigma_{i+1}) \in \tau \} \ .$$

We say that the transition relation τ generates the trace set $\mathcal{T}[\tau]$.

1. Give a definition for $\mathcal{S}[T]$, the function that, given a set of traces $T \subseteq \Sigma^+$, returns the smallest transition relation (for $\subseteq$) that generates a set of traces containing T.

2. Prove that the pair $\mathcal{S}$ and $\mathcal{T}$ forms a Galois connection between trace sets in $\mathcal{P}(\Sigma^+)$ and transition relations in $\mathcal{P}(\Sigma \times \Sigma)$:

$$(\mathcal{P}(\Sigma^+), \subseteq) \underset{\mathcal{S}}{\overset{\mathcal{T}}{\leftrightarrows}} (\mathcal{P}(\Sigma \times \Sigma), \subseteq)$$

3. Prove that not all trace sets are generated by a transition relation. Give an example where $\mathcal{S}$ results in an approximation.

4. Prove that the abstraction $\mathcal{S}[T]$ does not lose any information on T if and only if T is closed at the same time by junction ($T^\frown T = T$), by prefix, and by suffix, and if $\Sigma \subseteq T$.

Consider now the set $\mathcal{T}_\infty[\tau]$ of partial finite and infinite traces obeying τ:

$$\mathcal{T}_\infty[\tau] \stackrel{\text{def}}{=} \{ (\sigma_0, \ldots, \sigma_n) \in \Sigma^+ \mid \forall i < n : (\sigma_i, \sigma_{i+1}) \in \tau \} \cup \{ (\sigma_0, \ldots) \in \Sigma^\omega \mid \forall i : (\sigma_i, \sigma_{i+1}) \in \tau \}$$

5. Prove that it is possible to define a new Galois connection:

$$(\mathcal{P}(\Sigma^\infty), \subseteq) \underset{\mathcal{S}_\infty}{\overset{\mathcal{T}_\infty}{\leftrightarrows}} (\mathcal{P}(\Sigma \times \Sigma), \subseteq)$$

   by extending the function $\mathcal{S}$ from Question 1 to a function $\mathcal{S}_\infty : \mathcal{P}(\Sigma^\infty) \to \mathcal{P}(\Sigma \times \Sigma)$.

6. Prove that, for this new Galois connection, $\mathcal{S}_\infty[T]$ can lose some information on trace sets that are closed at the same time by prefix, by suffix, and by junction and contain Σ (give an example).

7. Provide a necessary and sufficient condition on T such that $\mathcal{S}_\infty[T]$ does not lose any precision in this new Galois connection.