Introduction

MPRI 2–6: Abstract Interpretation,
application to verification and static analysis

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Year 2023–2024

Course 0
14 September 2023

Formal Verification: Motivation

Historic example: Ariane 5, Flight 501

Maiden flight of the Ariane 5 Launcher, 4 June 1996.
Cost of failure estimated at more than 370 000 000 US$[1]

[1] M. Dowson. "The Ariane 5 Software Failure". Software Engineering Notes 22 (2): 84, March 1997.

Cause of Ariane 5 failure

Cause: software error[2]

- arithmetic overflow in unprotected data conversion from 64-bit float to 16-bit integer types[3]

```
P_M_DERIVE(T_ALG.E_BH) :=
  UC_16S_EN_16NS (TDB.T_ENTIER_16S
    ((1.0/C_M_LSB_BH) * G_M_INFO_DERIVE(T_ALG.E_BH)));
```

- software exception not caught
  $\Longrightarrow$ computer switched off
- all backup computers run the same software
  $\Longrightarrow$ all computers switched off, no guidance
  $\Longrightarrow$ rocket self-destructs

A "simple" error...

[2] J.-L. Lions et al., Ariane 501 Inquiry Board report.

[3] J.-J. Levy. Un petit bogue, un grand boum. Séminaire du Département d'informatique de l'ENS, 2010.

How can we avoid such failures?

- Choose a safe programming language.

  C (low level) / Ada, Java, OCaml (high level)

  yet, Ariane 5 software is written in Ada

- Carefully design the software.

  many software development methods exist

  yet, critical embedded software follow strict development processes

- Test the software extensively.

  yet, the erroneous code was well tested... on Ariane 4

$\Longrightarrow$ **not sufficient!**

How can we avoid such failures?

- Choose a safe programming language.

  C (low level) / Ada, Java, OCaml (high level)

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  many software development methods exist

  yet, critical embedded software follow strict development processes

- Test the software extensively.

  yet, the erroneous code was well tested... on Ariane 4

$\Longrightarrow$ **not sufficient!**

We should use **formal methods.**

provide rigorous, mathematical insurance of correctness
may not prove everything, but give a precise notion of what is proved

This case triggered the first large scale static code analysis

PolySpace Verifier, using abstract interpretation

Verification: compromises

Undecidability: correctness properties are undecidable!

no program can automatically and precisely separates all correct programs from all incorrect ones

Compromises: lose automation, or completeness, or soundness, or generality

- **Test, symbolic execution**: complete and automatic, but unsound
- **Theorem proving**
  - proof essentially manual, but checked automatically
  - powerful, but very steep learning curve and large effort required
- **Deductive methods**
  - automated proofs for some logic fragments (SAT, SMT)
  - still requires some program annotations (contracts, invariants)
- **Model checking**
  - check a (often hand-crafted) model of the program
  - finite or regular models, expressive properties (LTL)
  - automatic and complete (wrt. model)
- **Static analysis** (next slide)

Verification by static analysis

source

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    if (t[i]) break;
  }
  return t[i];
}
```

$\Longrightarrow$

analysis result

```
int search(int* t, int n) {
  int i;
  for (i=0; i<n; i++) {
    // 0 ≤ i < n
    if (t[i]) break;          ✓
  }
  // 0 ≤ i ≤ n ∨ n < 0
  return t[i];                ✗
}
```

- work directly on the source code
- infer properties on program executions
- automatically (cost effective)
- by constructing dynamically a semantic abstraction of the program
- to deduce program correctness, or raise alarms if it cannot
  implicit specification: absence of RTE; or (simple) user-defined properties: contracts
- with approximations (incomplete: efficient, but possible false alarms)
- soundly (no false positive)

Verification in practice: Example of avionics software

Critical avionics software is subject to certification:

- 70% of the development cost (in 2015)
- regulated by international standards (DO-178)
- mostly based on massive test campaigns & intellectual reviews

Current trend:

use of **formal methods** now acknowledged (DO-178C, DO-333)

- at the binary level, to replace testing
- at the source level, to replace intellectual reviews
- at the source level, to replace testing
  provided that the correspondence with the binary is also certified

$\Longrightarrow$ **formal methods can improve cost-effectiveness!**

Caveat: **soundness** is required by DO standards

Verification in practice: Formal verification at Airbus

Program proofs: deductive methods

- functional properties of small sequential C codes
- replace unit testing
- not fully automatic
- **Caveat / Frama-C** tool (CEA)

Sound static analysis:

- fully automated on large applications, non functional properties
- worst-case execution time and stack usage, on binary
  aiT, **StackAnalyzer** (AbsInt)
- absence of run-time error, on sequential C code
  Astrée analyzer (AbsInt)

Certified compilation:

- allows source-level analysis to certify sequential binary code
- **CompCert** C compiler, certified in **Coq** (INRIA)

Another example bug: Heartbleed

Vulnerability in OpenSSL cryptographic library
all versions from 2012 to 2014

OpenSSL is used by 66% of WEB servers for `https`
(also: email encryption, VPN, etc.)

Cause: buffer overflow in "heartbeat" protocol.

Consequence:[4]

- leak of private information, such as private keys
- no way to actually know what has been extracted
  $\Longrightarrow$ need to renew all keys after correcting the bug!
- very high economic cost!

[4] http://heartbleed.com

Improving software quality

Recent study from Consortium for Information & Software Quality:[5]

- $607 billions spent finding and fixing bugs
- $1.56 trillon cost for software failure
- just for 2020, just for the US!

⟹ even non-critical domains could use formal methods!

Challenges:

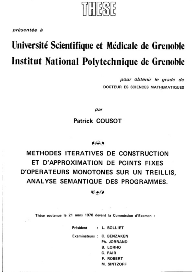
- keep up with scalability on critical software
- go beyond critical software (larger, more complex)
- more complex languages and programming models (C++, JavaScript, Python, ...)
- go beyond absence of run-time errors and towards functional properties
- while still being sound!

[5] Herb Krasner. The cost of poor software quality in the US: A 2020 report. https://www.it-cisq.org/pdf/CPSQ-2020-report.pdf, 2021. Accessed: 2021-08.

Overview of abstract interpretation

Abstract interpretation

Patrick Cousot[6]



THESE

présentée à

Université Scientifique et Médicale de Grenoble
Institut National Polytechnique de Grenoble

pour obtenir le grade de
DOCTEUR ES SCIENCES MATHEMATIQUES

par

Patrick COUSOT

METHODES ITERATIVES DE CONSTRUCTION
ET D'APPROXIMATION DE PCINTS FIXES
D'OPERATEURS MONOTONES SUR UN TREILLIS,
ANALYSE SEMANTIQUE DES PROGRAMMES.

Thèse soutenue le 21 mars 1978 devant la Commission d'Examen :

Président : L. BOLLIET

Examinateurs : C. BENZAKEN
Ph. JORRAND
B. LORHO
C. PAIR
F. ROBERT
M. SINTZOFF

General theory of the approximation and comparison of program semantics:

- unifies existing semantics
- guides the design of static analyses
  that are correct by construction

6. P. Cousot. "Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique des programmes." Thèse És Sciences Mathématiques, 1978.

Concrete collecting semantics

$(\mathcal{S}_0)$
```
assume X in [0,1000];
```
$(\mathcal{S}_1)$
```
I := 0;
```
$(\mathcal{S}_2)$
```
while (S3) I < X do
```
$(\mathcal{S}_4)$
```
    I := I + 2;
```
$(\mathcal{S}_5)$

$(\mathcal{S}_6)$

program

Concrete collecting semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I}, \mathtt{X}\} \to \mathbb{Z})$

$$
\begin{array}{lll}
\mathcal{S}_0 = \{\,(i,x) \mid i,x \in \mathbb{Z}\,\} & = \top \\
\mathcal{S}_1 = \{\,(i,x) \in \mathcal{S}_0 \mid x \in [0,1000]\,\} & = F_1(\mathcal{S}_0) \\
\mathcal{S}_2 = \{\,(0,x) \mid \exists i,\ (i,x) \in \mathcal{S}_1\,\} & = F_2(\mathcal{S}_1) \\
\mathcal{S}_3 = \mathcal{S}_2 \cup \mathcal{S}_5 & \\
\mathcal{S}_4 = \{\,(i,x) \in \mathcal{S}_3 \mid i < x\,\} & = F_4(\mathcal{S}_3) \\
\mathcal{S}_5 = \{\,(i+2,x) \mid (i,x) \in \mathcal{S}_4\,\} & = F_5(\mathcal{S}_4) \\
\mathcal{S}_6 = \{\,(i,x) \in \mathcal{S}_3 \mid i \geq x\,\} & = F_6(\mathcal{S}_3)
\end{array}
$$

semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{\mathtt{I}, \mathtt{X}\} \to \mathbb{Z})$:

- strongest program properties (inductive invariants)
- set of reachable environments, at each program point
- smallest solution of a system of equations
- well-defined solution, but not computable in general

Abstracting

Principle: be tractable by reasoning at an abstract level

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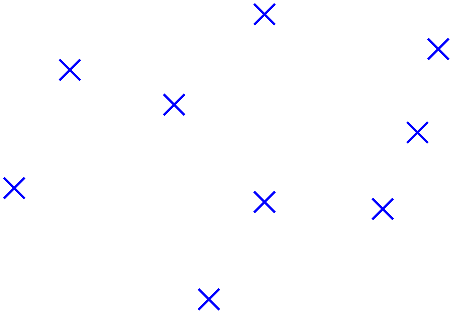
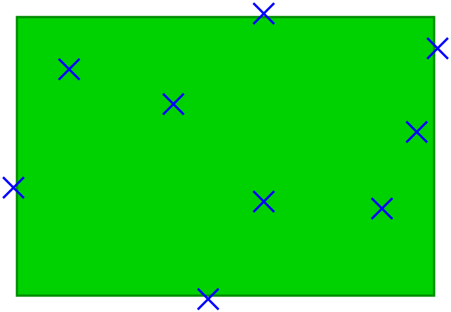


concrete executions : $\{(0,3),(5.5,0),(12,7),\ldots\}$ (not computable)

Abstracting

Principle: be tractable by reasoning at an abstract level

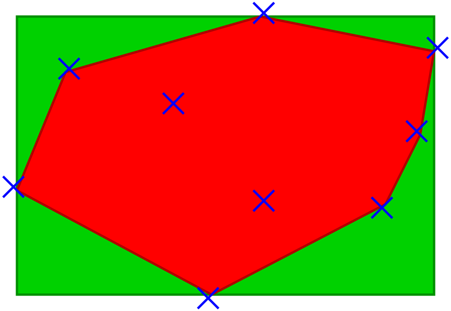


concrete executions : $\{(0,3),(5.5,0),(12,7),\ldots\}$ (not computable)

box domain : $X \in [0,12] \wedge Y \in [0,8]$ (linear cost)

Abstracting

Principle: be tractable by reasoning at an abstract level



concrete executions :	$\{(0,3),(5.5,0),(12,7),\ldots\}$	(not computable)
box domain :	$X \in [0,12] \wedge Y \in [0,8]$	(linear cost)
polyhedra domain :	$6X + 11Y \geq 33 \wedge \cdots$	(exponential cost)

many abstractions: trade-off cost vs. precision and expressiveness

From concrete to abstract semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$$\mathcal{S}_i \in \mathcal{D} \stackrel{\text{def}}{=} \mathcal{P}(\{I, X\} \to \mathbb{Z})$$

$$\begin{aligned}
\mathcal{S}_0 &= \{\, (i,x) \mid i,x \in \mathbb{Z} \,\} \\
\mathcal{S}_1 &= [\![\, X \in [0,1000] \,]\!]\,(\mathcal{S}_0) \\
\mathcal{S}_2 &= [\![\, I \leftarrow 0 \,]\!]\,(\mathcal{S}_1) \\
\mathcal{S}_3 &= \mathcal{S}_2 \cup \mathcal{S}_5 \\
\mathcal{S}_4 &= [\![\, I < X \,]\!]\,(\mathcal{S}_3) \\
\mathcal{S}_5 &= [\![\, I \leftarrow I + 2 \,]\!]\,(\mathcal{S}_4) \\
\mathcal{S}_6 &= [\![\, I \geq X \,]\!]\,(\mathcal{S}_3)
\end{aligned}$$

concrete semantics

Concrete semantics $\mathcal{S}_i \in \mathcal{D} = \mathcal{P}(\{I, X\} \to \mathbb{Z})$:

- $[\![\, X \in [0,1000] \,]\!]$, $[\![\, I \leftarrow 0 \,]\!]$, etc. are transfer functions
- strongest program properties
- set of reachable environments, at each program point
- not computable in general

From concrete to abstract semantics

```
(S0)
assume X in [0,1000];
(S1)
I := 0;
(S2)
while (S3) I < X do
    (S4)
    I := I + 2;
    (S5)
(S6)
```

program

$$
\begin{aligned}
&\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp \\
&\mathcal{S}_0^\sharp = \top^\sharp \\
&\mathcal{S}_1^\sharp = [\![\, X \in [0,1000] \,]\!]^\sharp(\mathcal{S}_0^\sharp) \\
&\mathcal{S}_2^\sharp = [\![\, I \leftarrow 0 \,]\!]^\sharp(\mathcal{S}_1^\sharp) \\
&\mathcal{S}_3^\sharp = \mathcal{S}_2^\sharp \cup^\sharp \mathcal{S}_5^\sharp \\
&\mathcal{S}_4^\sharp = [\![\, I < X \,]\!]^\sharp(\mathcal{S}_3^\sharp) \\
&\mathcal{S}_5^\sharp = [\![\, I \leftarrow I + 2 \,]\!]^\sharp(\mathcal{S}_4^\sharp) \\
&\mathcal{S}_6^\sharp = [\![\, I \geq X \,]\!]^\sharp(\mathcal{S}_3^\sharp)
\end{aligned}
$$

abstract semantics

Abstract semantics $\mathcal{S}_i^\sharp \in \mathcal{D}^\sharp$:

- $\mathcal{D}^\sharp$ is a subset of properties of interest
  semantic choice + machine representation
- $F^\sharp : \mathcal{D}^\sharp \to \mathcal{D}^\sharp$ over-approximates the effect of $F : \mathcal{D} \to \mathcal{D}$ in $\mathcal{D}^\sharp$
  abstract operators proved sound + effective algorithms

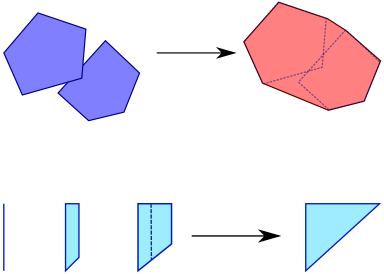
Abstract operator examples

In the polyhedra domain:

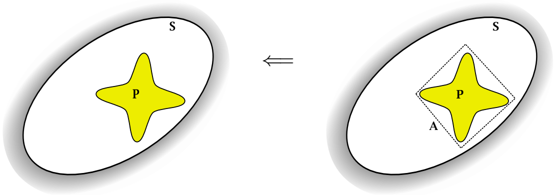
- Abstract assignment
  $[\![X \leftarrow X + 1]\!]^\sharp$
  translation (exact)

- Abstract union
  $\cup^\sharp$
  convex hull (approximate)

- Solving the equation system
  by iteration
  using extrapolation to terminate



Soundness and false alarms



$P \subseteq S$

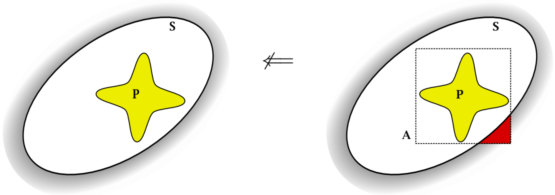
$A \subseteq S$
program proved

Goal: prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

Soundness and false alarms



$P \subseteq S$ $\quad\not\Longleftarrow\quad$ $A \not\subseteq S$
false alarm

<u>Goal:</u> prove that a program P satisfies its specification S

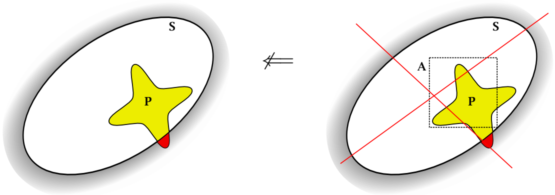
We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

Soundness and false alarms



$P \not\subseteq S$ $\not\Leftarrow$ $A \subseteq S$
false negative
cannot occur

Goal: prove that a program P satisfies its specification S

We collect the reachable states P and compare to S

A polyhedral abstraction A can prove the correctness

A box abstraction cannot prove the correctness
$\Longrightarrow$ false alarm

(especially since the analysis may not output the tightest box / polyhedron!)

The analaysis is sound: no false negative reported!

Getting it right? eBPF example

eBPF:

- a virtual machine inside the Linux kernel
- can run arbitrary code in kernel mode
- very low-level, can perform arbitrary pointer arithmetic (flat memory model)
- run sandboxed to protect agains bugs and attacks

In theory:

- a static analysis checks bytecode safety before execution
- includes an interval analysis for pointers

Getting it *not* right! eBPF example

Bound computation for bit-shift >>:[7]

```
case BPF_RSH:
  if (min_val < 0 || dst_reg->min_value < 0)
    dst_reg->min_value = BPF_REGISTER_MIN_RANGE;
  else
    dst_reg->min_value = (u64)(dst_reg->min_value) >> min_val;
  if (dst_reg->max_value != BPF_REGISTER_MAX_RANGE)
    dst_reg->max_value >>= max_val;
  break;
```

A dynamic analysis has been added...
which exploits the (unsound) results from the static analysis...

Lesson

Use abstract interpretation to make analyses sound by construction!

[7] https://www.zerodayinitiative.com/blog/2021/1/18/zdi-20-1440-an-incorrect-calculation-bug-in-the-linux-kernel-ebpf-verifier

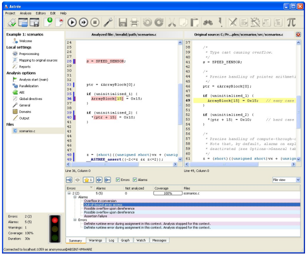
Example tools

Astrée

Astrée: developed at ENS & INRIA by P. Cousot & al.

- analyzes embedded critical C software
  subset of C, no memory allocation, no recursivity → simpler semantics
- checks for run-time errors
  arithmetic overflows, array overflows, divisions by 0, pointer errors, etc. → non-functional
- specialized for control / command software
  with zero false alarm goal
  application domain specific abstractions

Airbus A380



2001–2004: academic success
proof of absence of RTE
on flight command

2009: industrialization

AbsInt

Infer.AI

Infer: http://fbinfer.com/

- developed at Facebook (team formerly at Monoidics)
- Infer.AI is an analysis framework based on abstract interpretation
- open-source since 2015
- analyzes Java, C, C++, and Objective-C
- checks ThreadSafety (Java), Initalisation Order (C++), etc.
- modular, bottom-up interprocedural analysis
- targets the analysis of merge requests (small bits at a time)
- favors speed over soundness
  pragmatic choices, based on "what engineers want"
  no requirements for certification, unlike the avionics industry...
- used in production

Frama-C

Frama-C: https://frama-c.com/

- developed at CEA
- open-source
- analyzes C
- combines abstract interpretation and deductive methods
- has a specification language (ACSL) for functional verification
- used in industrial applications

Example research project: MOPSA

Modular Open Platform For Static Analysis
developed at Sorbonne University: https://mopsa.lip6.fr/

An abstract interpreter prototype tool for research and education

- extendable to new properties and new languages
- help developing, reusing, combining abstractions
- open-source: https://gitlab.com/mopsa/mopsa-analyzer

Currently available: (not fully scalable!)

- C analysis for run-time error detection
- Python analysis (supports a large subset of Python 3, and a small subset of its library)
- analysis of programs mixing C and Python

On-going research: (not public yet, various level of maturity)

- patch and portability analysis for C
- analysis of smart-contracts (Michelson language for the Tezos blockchain)
- security-related properties

Course organisation

Teaching team

Caterina Urban

Jérôme Feret

Antoine Miné

Xavier Rival

Syllabus and exams

https://www-apr.lip6.fr/~mine/enseignement/mpri/2023-2024

Visit regularly for:

- latest information on course dates and modalities
  and, possibly, last-minute changes
- course material (slides)
- optional course assignments and reading
- internship proposals

Exams:

- 50%: written mid-term exam (3h)
- 50%: oral final exam
  (read a scientific article, present it, answer questions)

Course material

Available on the web page:

- main material: slides

- course notes

  cover mainly foundations and numeric abstract domains
  based on:

  A. Miné. *Tutorial on Static Inference of Numeric Invariants by Abstract Interpretation*. In Foundations and Trends in Programming Languages, 4(3–4), 120–372. Now Publishers.

- recommended reading on theory and applications:

  J. Bertrane, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, X. Rival. *Static analysis and verification of aerospace software by abstract interpretation.* In Foundations and Trends in Programming Languages, 2(2–3), 71–190, 2015. Now Publishers.

Course assignments (self-evaluation)

On the web page, **recommended** homework

- exercises: prove a theorem, solve a former exam, etc.
- reading assignments: read an article related to the course
- experimentation: use a tool

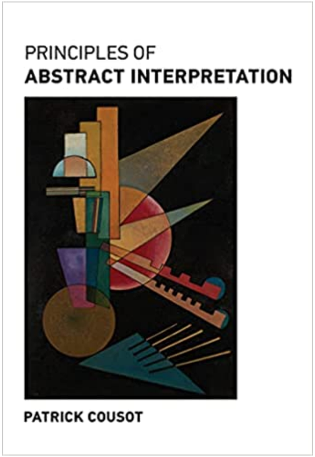
Also:

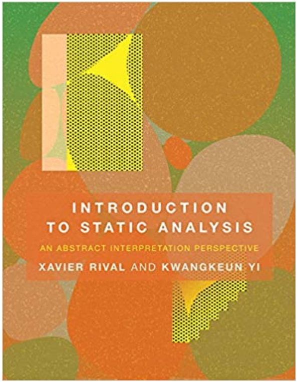
- previous exams, some with correction
- example programming project (in French)
  (abstract interpreter for a toy language in OCaml)

Principle: self-evaluation

- no credit
- not corrected by the teachers

Recent books!



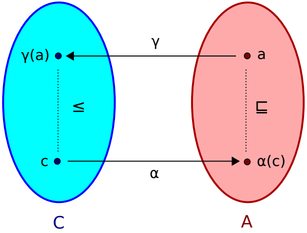


1. P. Cousot. Principles of Abstract Interpretation. 832 pages. The MIT Press. Sept. 2021.
2. X. Rival and K. Yi. Introduction to Static Analysis: An Abstract Interpretation Perspective. 320 pages. The MIT Press. Feb, 2020.

Course plan (1/8)

Foundations of abstract interpretation: (courses 1 & 2)

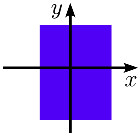
- mathematical background: order theory and fixpoints
- formalization of abstraction, soundness
- program semantics and program properties
- hierarchy of collecting semantics



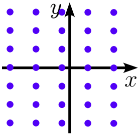
Course plan (2/8)

Bricks of abstraction: numerical domains

simple domains

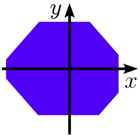


Intervals
$x \in [a, b]$

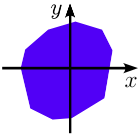


Congruences
$x \in a\mathbb{Z} + b$

relational domains

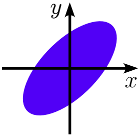


Octagons
$\pm x \pm y \leq c$

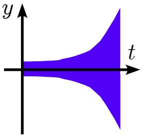


Polyhedra
$\sum_i \alpha_i x_i \leq \beta$

specific domains



Ellipsoids
digital filters

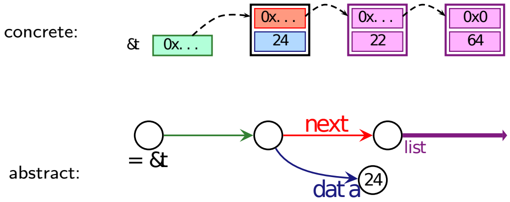


Exponentials
rounding errors

Course plan (3/8)

Bricks of abstraction: memory abstractions

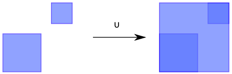
- beyond numeric: reason on arrays, lists, trees, graphs, . . .
- challenges: variety of structures, destructive updates
- logical tools:
  - separation logics (a logic tailored for describing memory)
  - parametric three valued logics (representing arbitrary graphs)
- abstract domains based on these logics



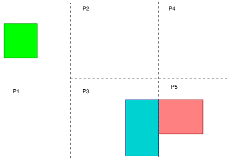
Course plan (4/8)

Bricks of abstraction: partitioning abstractions

- most abstract domains are not distributive
  $\Longrightarrow$ reasoning over disjunctions loses precision
- first solution: add disjunctions to any abstract domain
  $\Longrightarrow$ expressive but costly
- second solution: partitioning
  conjunctions of implications as logical predicates
  (partitioning may be based on many semantic criteria)



loss of precision



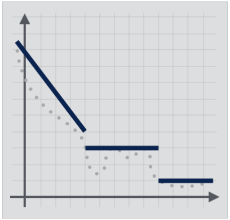
partitioning

Course plan (5/8)

Analyses: abstract interpretation for liveness properties

- beyond safety (e.g., absence of errors)
  we prove that programs (eventually) do something good

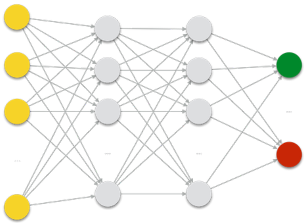
- abstract domains to reason about program termination
  inference of ranking functions



- generalization to other liveness properties
  (e.g., expressed in CTL)

Course plan (6/8)

Analyses: static analysis of neural networks

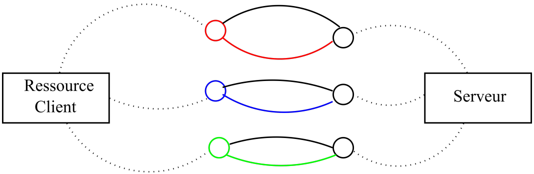


- verification of local robustness against adversarial examples
- fairness certification
  (special case of global robustness verification)
- verification of functional properties

Course plan (7/8)

Analyses: analysis of mobile systems

- dynamic creation of components and links
- analyze the links between components
  - distinguish between recursive components
  - abstractions as sets of words
- bound the number of components
  using numeric relations



Course plan (8/8)

Analyses: static analysis for security

- challenge: security properties are diverse
  from information leakage to unwanted execution of malicious code
  and more complex than safety and liveness
- the framework of hyperproperties can express security
- apply abstract interpretation to reason over non-interference

Internship proposals

Possibility of Master 2 internships at ENS, Sorbonne Université or INRIA Lille.

Example topics:

- Incremental static analysis of evolving software
- Static analysis for multi-language programs
- Static analysis of smoothness properties
- Determining the impact of vulnerabilities using semantic dependencies
- Static analysis under a time budget
- Static analysis of the robustness of machine-learning software
- Abstract domain reductions between separation logic and value abstractions
- ...

Formal proposals will be available on the course page
also: discuss with your teachers for tailor-made subjects.