

Best-of-two-worlds analysis of online search

Spyros Angelopoulos

Christoph Dürr

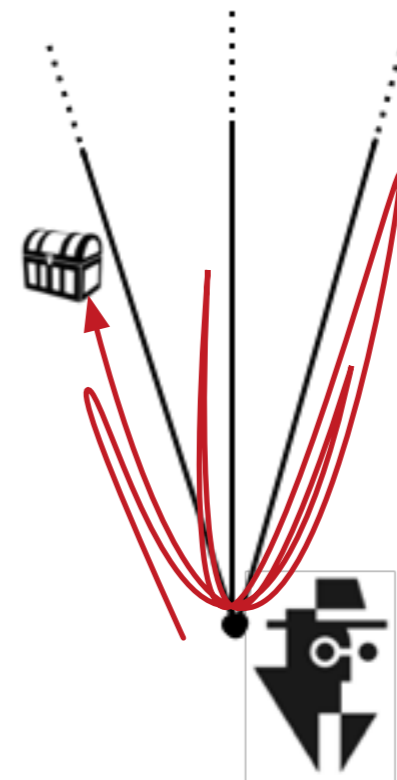
Shendan Jin

(Sorbonne Université, LIP6)

spyros is really the initiator
and should have been
given the talk instead

Online search

- Search space with an origin
- **Immobile hider:**
at unknown position, (technical assumption: distance at least 1 from origin)
- **Mobile searcher:**
initially at origin. Many cost models (turning cost, exploring known area is free)
- Question:
how to **measure efficiency**
of a search strategy ?



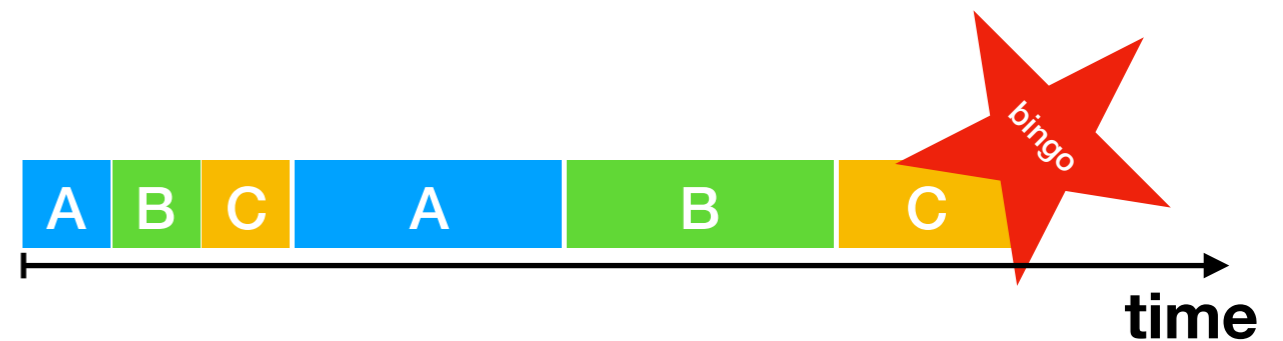
Application: demining

- You know: A single mine is set in some crossing of a road network
- You don't know: where
- Demining a road segment takes some time
- Walking along demined roads takes no time
- In what order should you proceed ?



Application: scheduling heuristics

- You are given a problem instance I
- and a fixed number of heuristics for this problem, with the promise that one will eventually succeed on I. But you don't know which one, and after how many CPU cycles.



Considered variant: cow path problem:



$y-x_3$

$-x_1$

0

x_0

x_2

x_4

- there is juicy grass on the other side of the fence
- cow is at position 0,
fence has an opening at position y with $|y| \geq l$
($\text{sign}(y)$ is unknown to the cow)
- strategy: $l \leq x_0, x_1, x_2, x_3, \dots$ with $x_i \geq x_{i-2}$
- **Standard measure:**
competitive ratio := distance walked by cow / $|y|$

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- strategy: $l \leq x_0, x_1, x_2, x_3, \dots$ with $x_i \geq x_{i-2}$
- worst case: $y = x_n + \varepsilon$ for some n
- distance walked by cow = $2(x_0 + \dots + x_{n+1}) + x_n + \varepsilon$
- *competitive ratio* is at least 9
doubling strategy ($x_i = 2^i$) achieves 9

Problem has long history

Competitive ratio

- Initially proposed by Bellman and Beck (1963) in a Bayesian context
- First solved by Beck and Newman : optimal competitive ratio of 9
- Many papers by Beck in 70's, 80's, 90's
- [Gal 72, 74] : solution to the m-ray problem
- Rediscovered in [Baeza-Yates et al, 96]
- Many variants :
 - Multiple searchers [Lopez-Ortiz and Schuierer 02]
 - Turn cost [Demaine et al 04], [Angelopoulos et al 17]
 - Randomized strategies [Kao et al 96]
 - Probabilistic search [Jaillet and Stafford 01]
 - Weighted targets [Angelopoulos and Panagiotou 16]
 - Distance bounds [Bose et al 13] and many others

Set of optimal strategies

Competitive ratio

- Characterization of search strategies with optimal competitive ratio

$$1 \leq x_0 \leq 4, \quad x_1 \geq 1 \quad \text{and} \quad x_n \leq 3x_{n-1} - \sum_{i=0}^{n-2} x_i, \quad \text{for all } n \geq 1.$$



doubling $x_i=2^i$



agressive $x_i=4(i+1)2^i$

- Situation similar to the paging problem, where LRU and say FIFO have the same optimal ratio. Motivated the study of *bijective analysis*.

Alternative measures

for online algorithms in general

Max/Max ratio [Ben-David, Borodin 94]

Loose Competitiveness [Young 94]

Diffuse Adversary [Koutsoupias Papadimitriou 00]

Random Order Ratio [Kenyon 96]

Relative Worst Order Ratio [Boyar Favrholdt 03]

Paging with **locality of reference**

Access Graph Model [BIRS 95]

Concave Analysis [Albers, Favrholdt, Giel 05]

Adequate Analysis [Panagiotou, Souza 06]

[Dorrigiv Lopez-Ortiz 05]: A survey of Performance Measures for On-line Algorithms. ACM SIGACT News

Our new measure



y

0

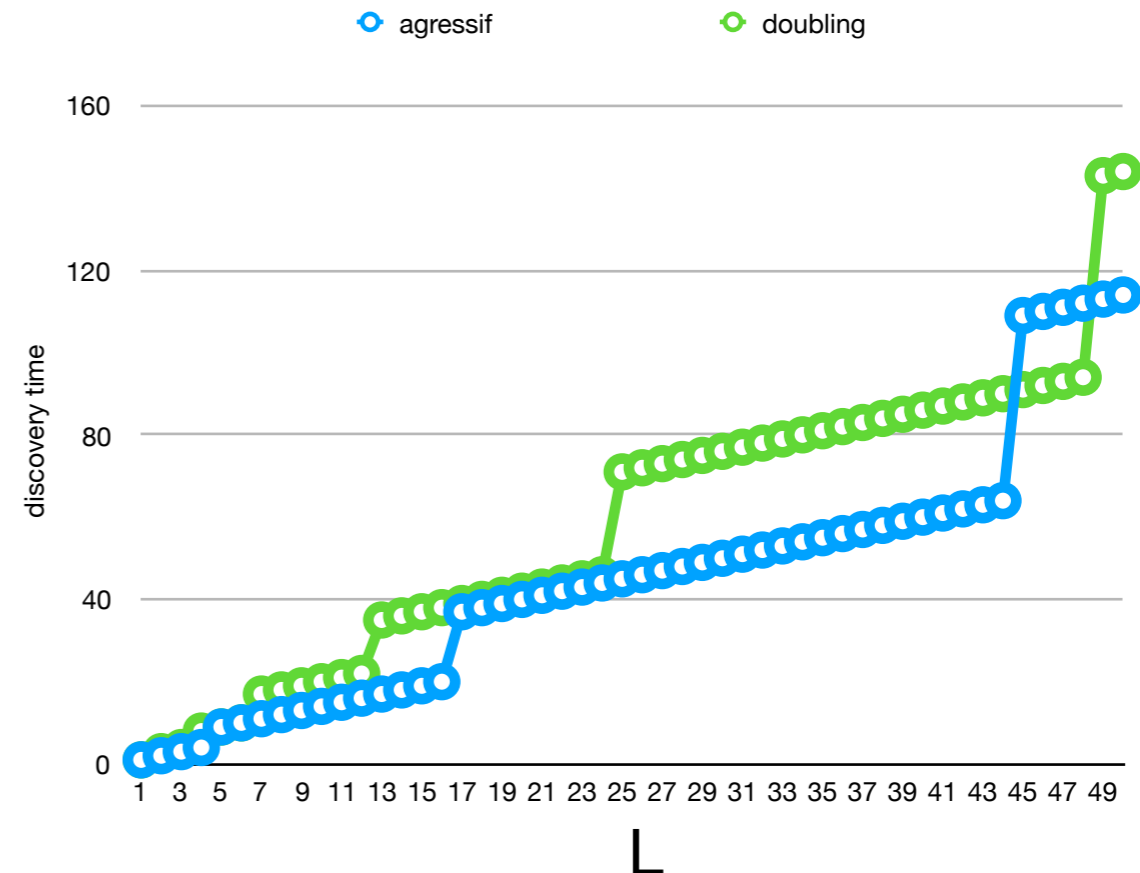
L

- **Discovery time**
 $D(x,L) :=$ time it takes for strategy x to discover a segment of length L
- Intuitively we prefer strategies with *small* discovery times. Inversely they *explore more* in the same time.
- Should help us to distinguish among optimal strategies with respect to the competitive ratio



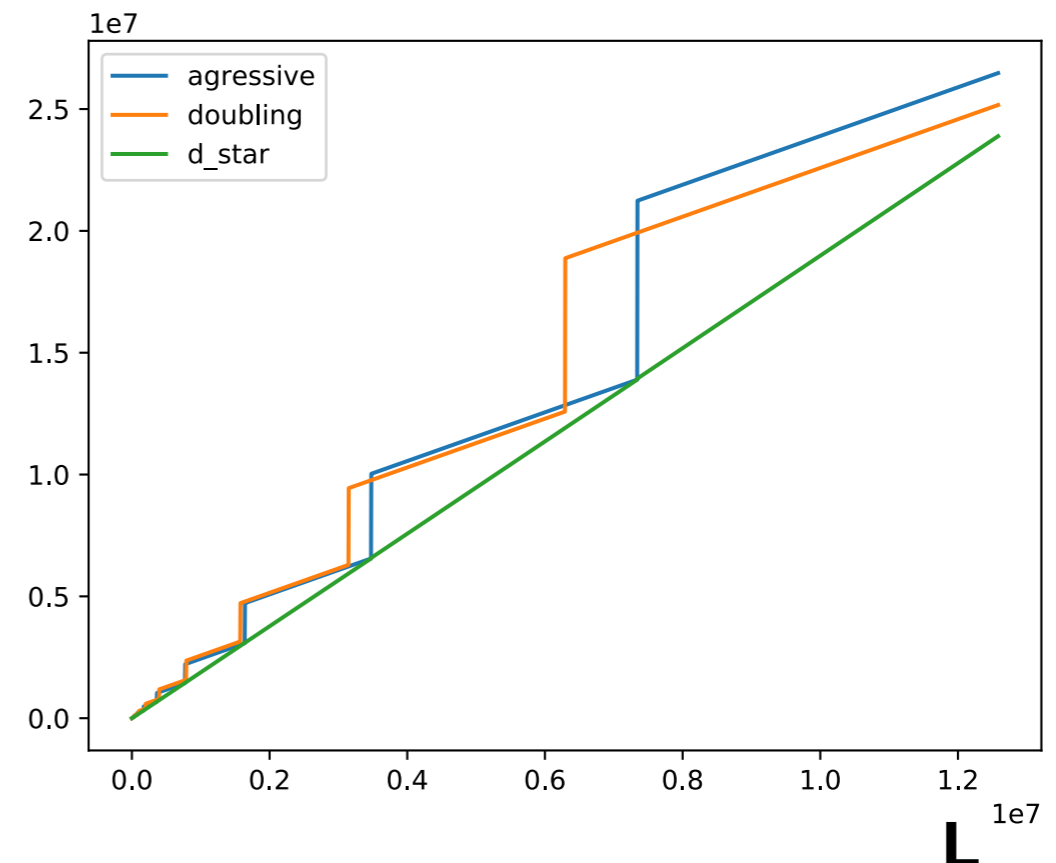
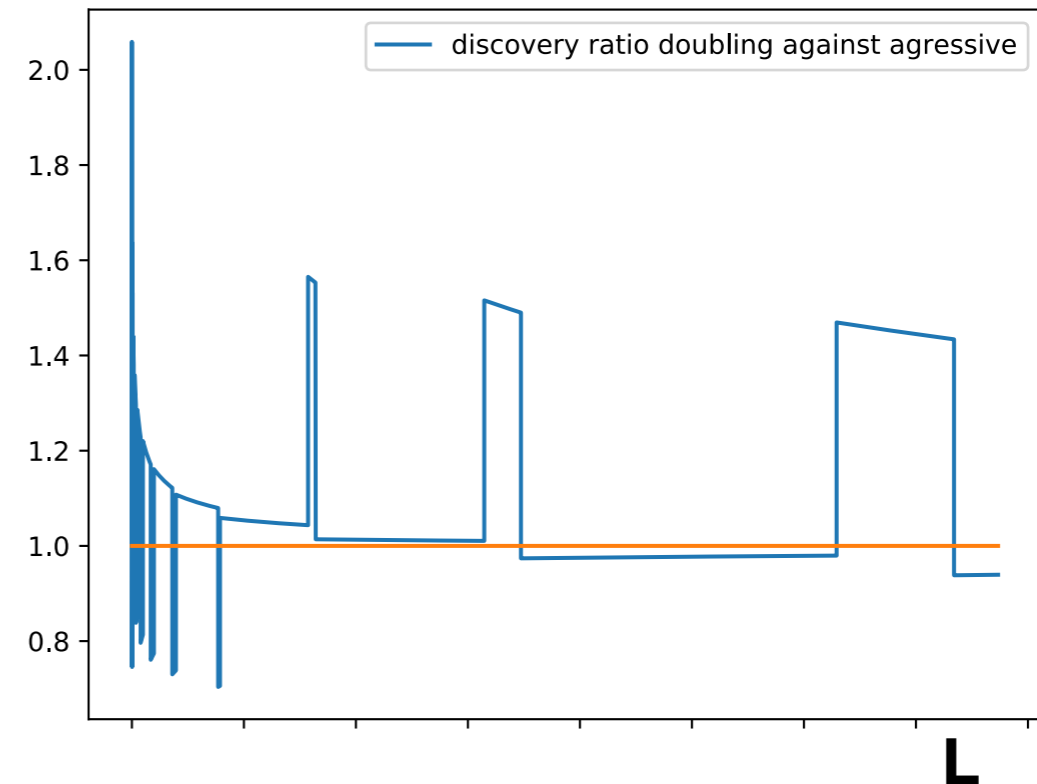
How to compare two strategies under this measure ?

- In this example *agressif* seems to dominate *doubling*, but not for all lengths L .
- What happens for $L \in [44, 48]$, is it seldom?



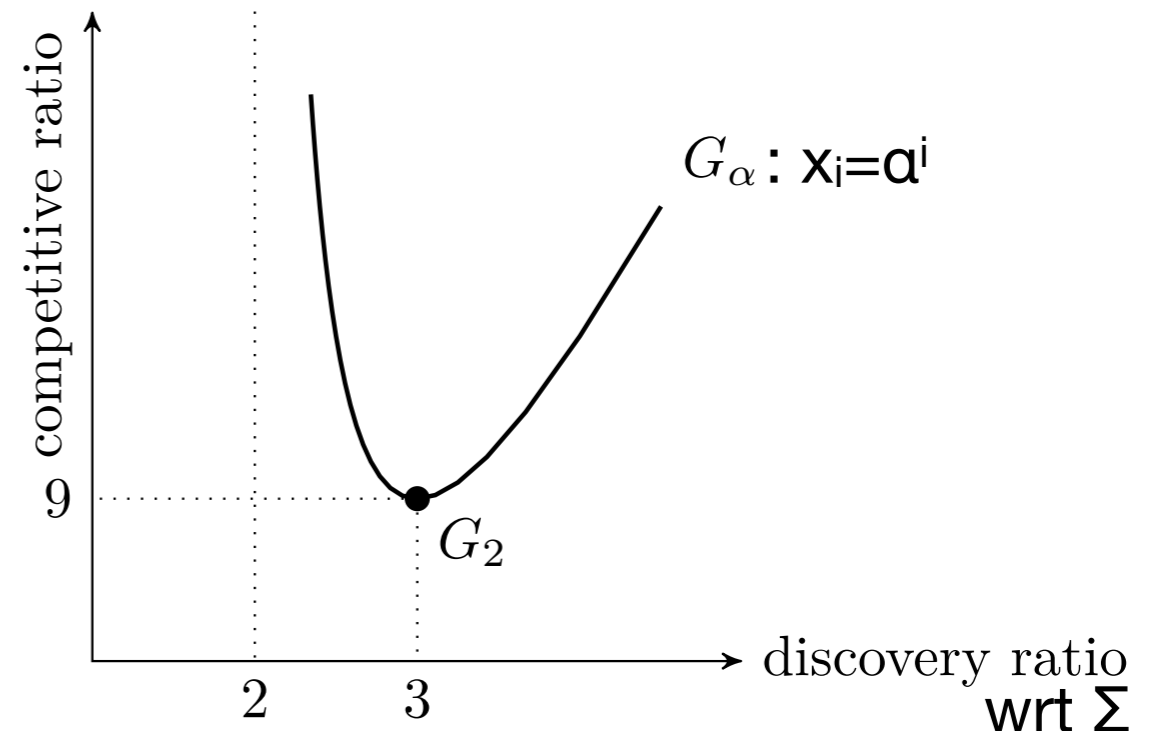
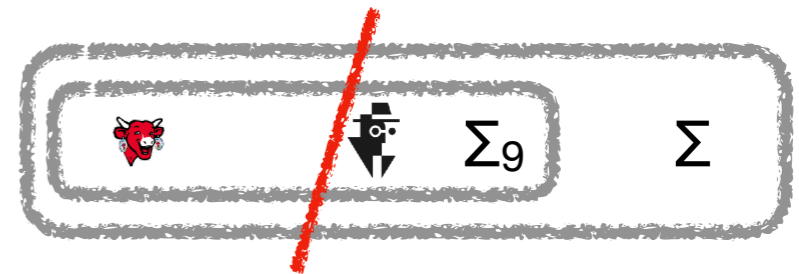
No

- $d^*(L)$ is the optimal discovery time among strategies in Σ_9 . It is reached by the strategy $x_i = t(i+1)2^i$ for some optimized t .
- There is no strategy in Σ_9 that is optimal for *all* L .
- S = set of strategies, either all (Σ), or only strategies with optimal competitive ratio (Σ_9)
- Denote $D(x,L) :=$ discovery time $:=$ time for x until it discovers a segment of length L
- Discovery ratio of a given strategy x against y is the ratio $D(x,L) / D(y,L)$ maximized over L
- **Discovery ratio $DR(x,S)$** of a given strategy x is the ratio of x against y , maximized over all strategies $y \in S$.



Our results

- **Discovery ratio $DR(x, S)$** of a given strategy x is the ratio of x against y , maximized over all strategies $y \in S$.
- **Results:**
 $DR(x, \Sigma_9) \geq 8/5$ for all $x \in \Sigma_9$
 $DR(\text{aggressive}, \Sigma_9) = 8/5$
 $DR(\text{doubling}, \Sigma_9) = 7/3$
- If $DR(x, \Sigma_9) = 8/5$, then x coincides with aggressive on the first 5 steps, but not always on the 6th



thank you