

# Let's play a game

All of you write a sequence of increasing numbers from 1 ending at 100.

Then I will reveal a target  $u$

Your cost is the sum of the values in the sequence, until the first number being at least  $u$ .

Your competitive ratio is  $\frac{\text{cost}}{u}$ .

Who will have the smallest ratio?

example

2 3 5 17 72 100

$\underbrace{\quad\quad\quad}_6$   
27

$$\frac{27}{6} = 4.5$$

This game is called

# Online Bidding

2

and has  
been introduced  
by Marek Chrobak  
Claire Mathieu  
in 2006

formally

find a sequence  $x = (x_i)_{i \in \mathbb{Z}}$

minimizing  $\sup_u \frac{\text{cost}(x, u)}{u}$

for  $\text{cost}(x, u) := \sum_{i \leq j} x_i$

where  $j$  is the smallest  
index s.t.  $x_j \geq u$ .

State of the art ( $\exists$  matching lower bounds)

deterministic ratio = 4  
is obtained by doubling

$$x_i = 2^i$$

randomized ratio =  $e \approx 2.718$   
 $\frac{\mathbb{E}[\text{cost}(x, u)]}{u}$

is obtained by random shift

$$x_i = e^{i+\lambda} \quad \lambda \in U[0, 1]$$

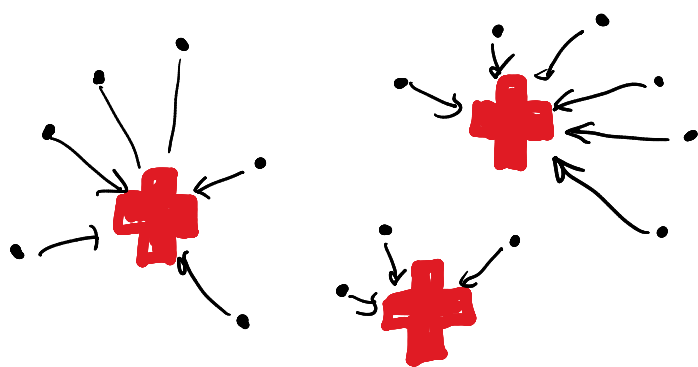
# Applications

3

The cow path problem aka linear search



Incremental Medians

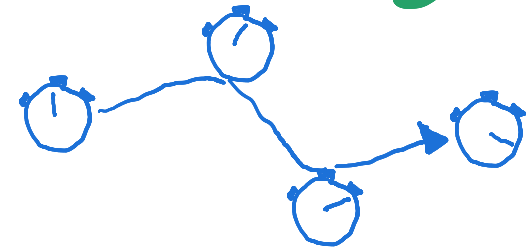


build  $n$  hospitals one by one  
after  $k$  hospitals you run out of money  
↑  
not known in advance

cost = total distance to closest hosp.

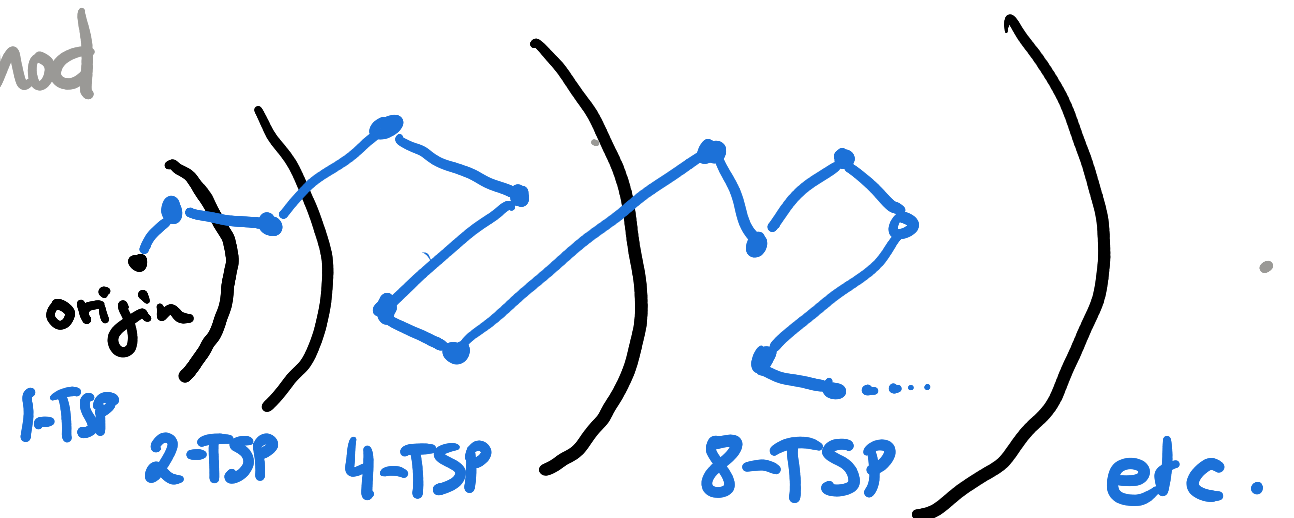
ratio = cost / optimal cost if  $k$  was known in advance.

Minimum Latency Tour



input:  $n$  points in metric space  
output: tour visiting all points  
cost: total latency (time of visit)

method



$k$ -TSP = (approximately) shortest  
traveling salesperson tour  
visiting  $k$  points

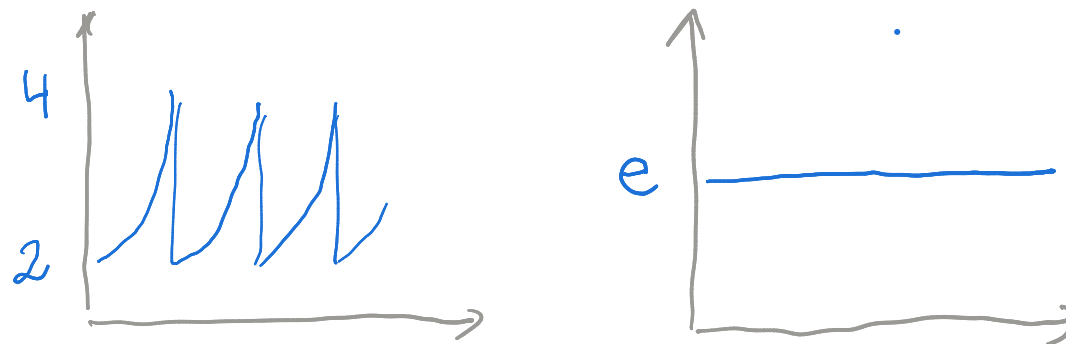
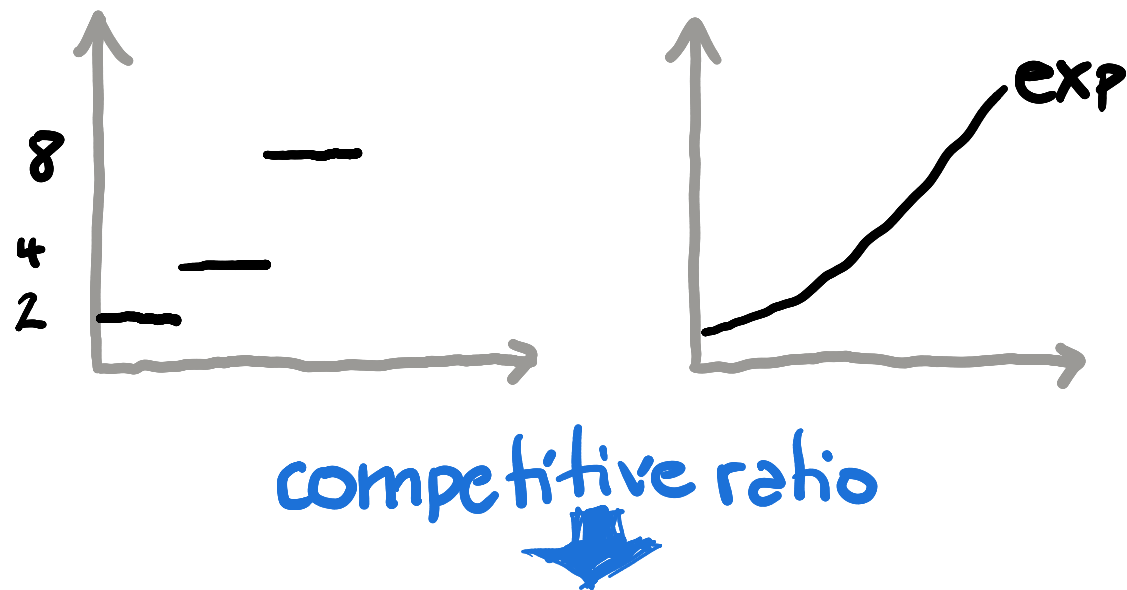
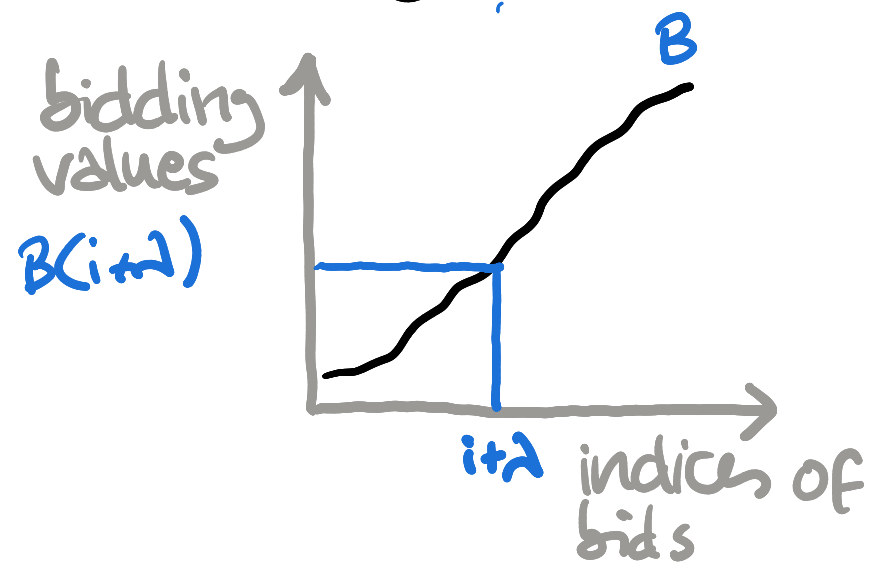
[Goemans  
kleinberg 1996]

# Our Contribution

4

Represent bidding sequences  
by bidding functions

needs some exchange argument



# Our Contribution

[Desryse, Zaakour, Angelopoulos, D, 2026]

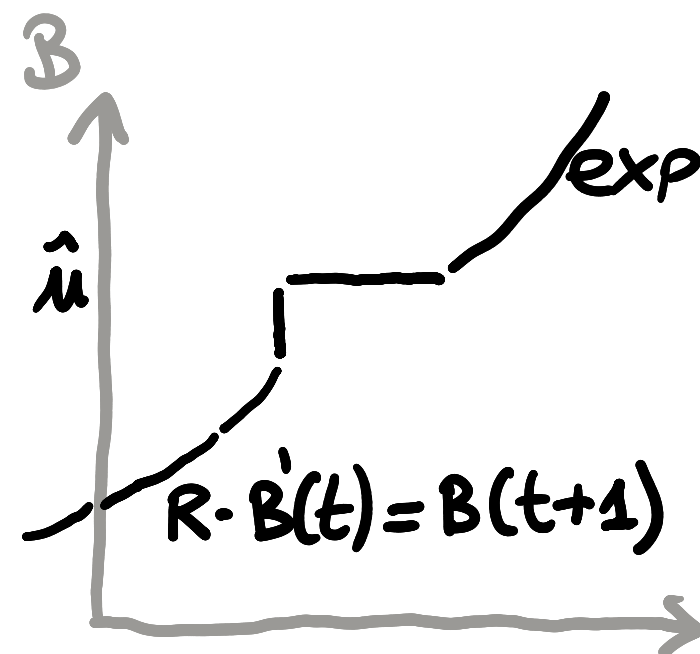
Algorithm obtains a prediction  $\hat{u}$  for the hidden target  $u$

Consistency = competitive ratio in case  $\hat{u} = u$

Robustness = competitive ratio in the worst case



Obtained  
by optimizing  
the bidding  
function



result obtained  
independently  
by  
[Lee, Lee, Lee, Shin, Yun]

# Thank you

for your attention

