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Set Selection with Uncertain Weights: Non-Adaptive Queries and Thresholds

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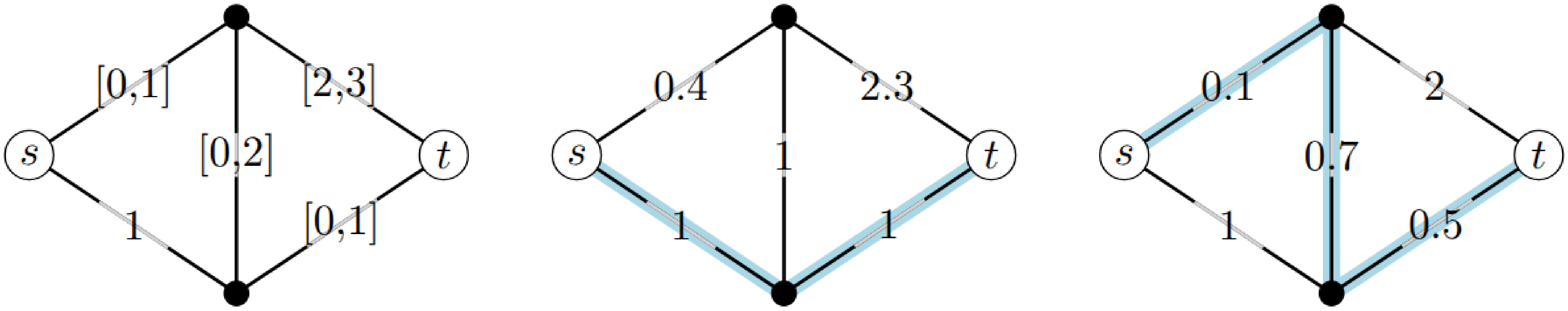
What is your favorite combinatorial optimization problem ?

s-t shortest path (good choice, good choice)

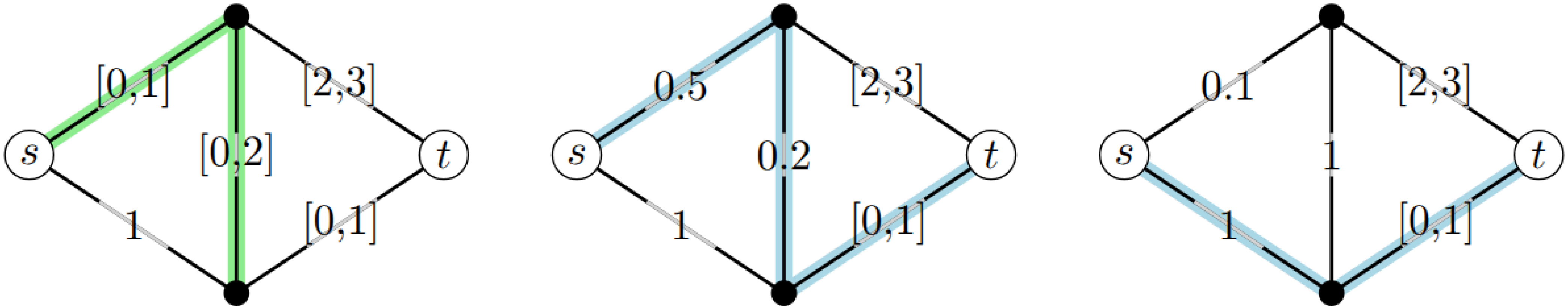
- **input:** graph $G = (V, E)$, every $e \in E$ has uncertain edge weight $w_e \in [\ell_e, u_e]$
- **output:** *universal* shortest $s - t$ path: which is optimal for every $w \in [\ell, u]$



This example does not have a universal optimal solution

Admissible query set

- **input:** graph $G = (V, E)$, every $e \in E$ has uncertain edge weight $w_e \in [\ell_e, u_e]$
- **output:** Minimum cardinality $Q \subseteq E$, such that if for every $e \in Q$ the uncertain interval $[\ell_e, u_e]$ is replaced by some value $w_e \in [\ell_e, u_e]$, then a universal shortest $s - t$ path exists.

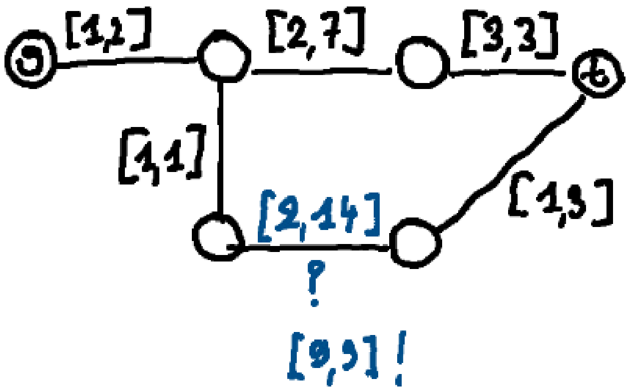


After querying all edges from Q an universal optimal solution exists

Connection with explorable uncertainty

Explorable uncertainty

- Weights belong to intervals
- Query of a weight reduces it to a singleton
- Make minimum number (or total cost) of queries until *universal* optimal solution can be identified



Connection

- No edge needs to be queried iff for every edge e the interval (ℓ_e, u_e) is disjoint from $[T_e^+, T_e^-]$. (⭐)

Hence if we have an oracle for (⭐) then we can compute $[T_e^+, T_e^-]$ using binary search

How to compute an admissible query set ?

Adaptive

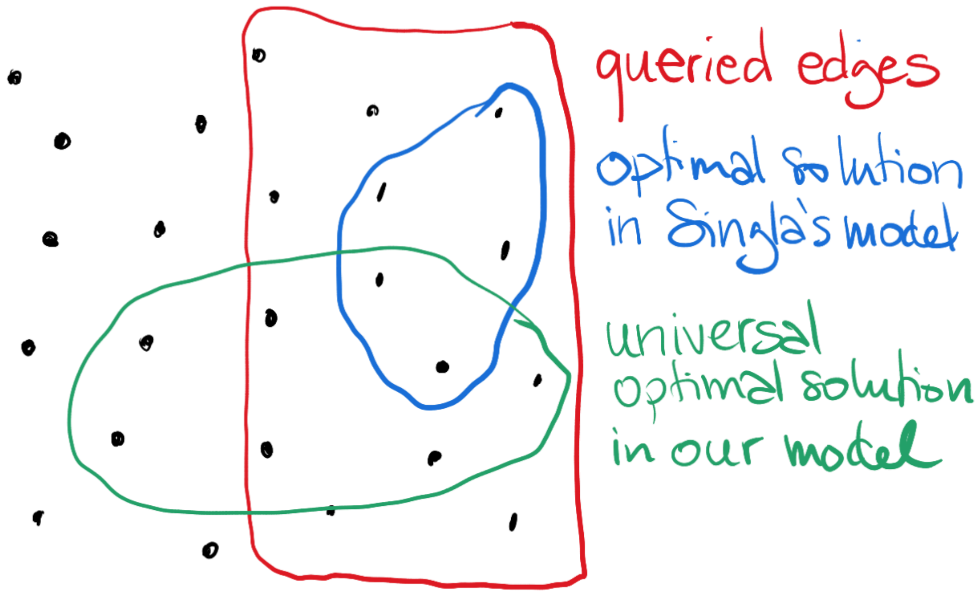
- [Kahan 1991]
- [Erlebach, ea 2008] (2-approximation for minimum spanning tree)
- ...

$\neq$ [Singla 2017] Price of Information

- Different model: make minimum number of queries such that optimal solutions **consists of** only queried items

Non-adaptive

- [Merino, Soto 2019] Polynomial time for minimum spanning tree



Threshold (folklore)

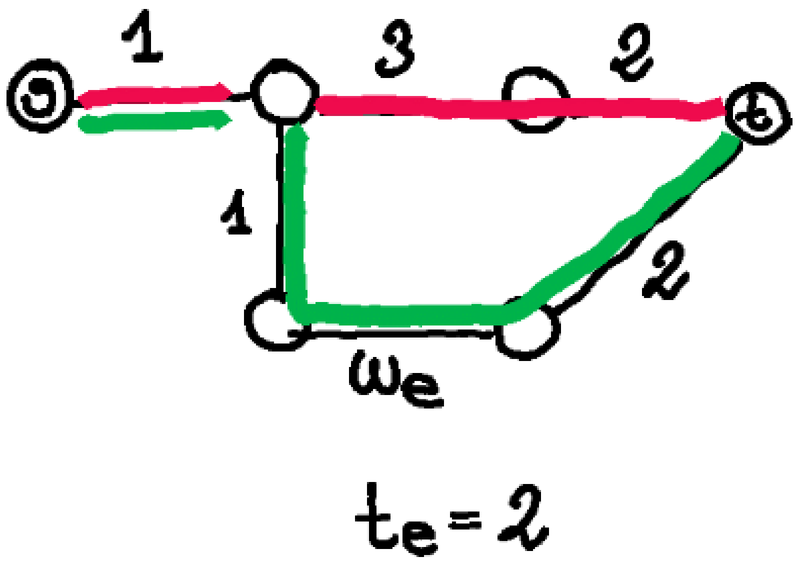
Model without uncertainty

- **input:** graph $G = (V, E)$, edge weights w_f for all $f \in E$
- **fixed edge:** $e \in E$
- **output:** T_e such that
  - if $w_e < T_e$, e belongs to every shortest path
  - if $w_e > T_e$, e does not belong to any shortest path
  - if $w_e = T_e$, There are shortest paths with and without e

T_e is function of the other weights w_{-e}

Trivial to compute

- T_e is difference of shortest $s - t$ path without e and shortest $s - t$ path with e with $w_e := 0$

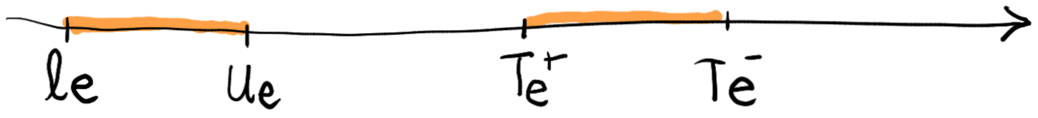


Thresholds of inclusion and of exclusion

Consider an edge e

there are T_e^+, T_e^-, depending on ℓ_{-e}, u_{-e} such that

- if $u_e < T_e^+$, e belongs to every $s-t$ shortest path, for every weight realization $w \in [\ell, u]$
- if $\ell_e > T_e^-$, e does not belong to any $s-t$ shortest paths
- if $[\ell_e, u_e]$ and $[T_e^+, T_e^-]$ intersect, there are $s-t$ shortest paths with and without e



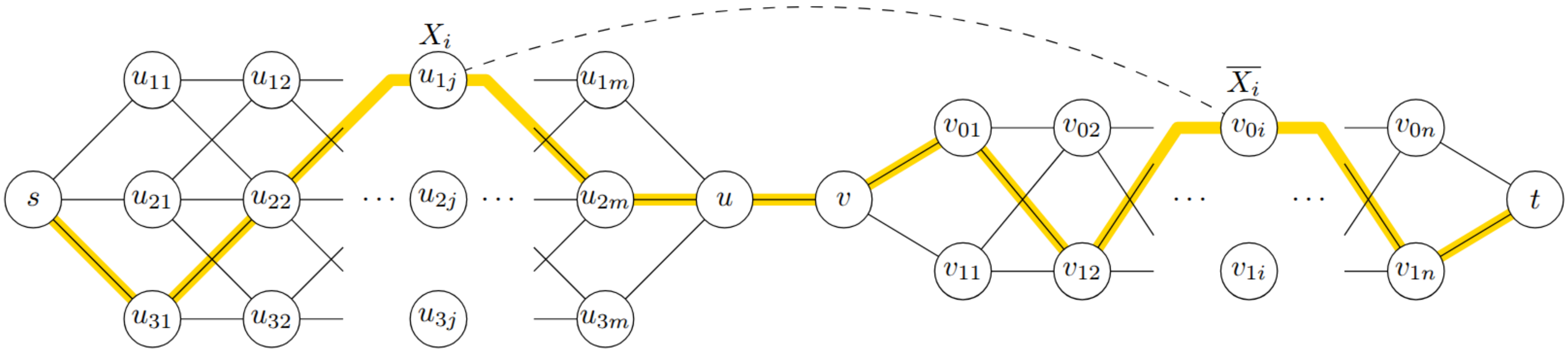
Can we compute these thresholds?

- For minimum spanning tree in time $O(E\alpha(E,V)+V)$
- For maximum weight matching on trees in linear time
- For shortest path on a DAG it is NP-hard
- For min cost bipartite perfect matching it is NP-hard

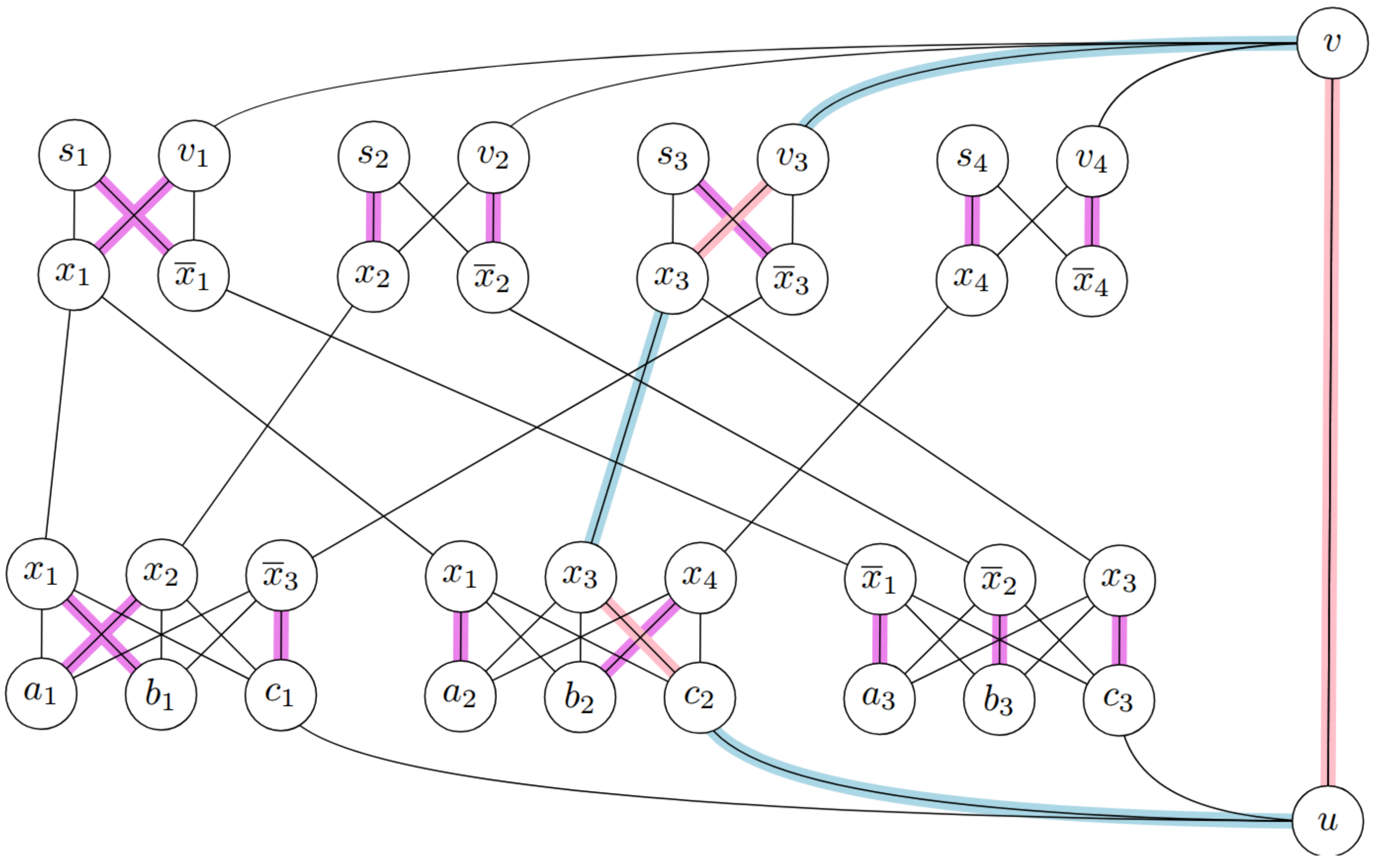
NP-hardness for shortest path

Adaptation from [Gabow, Maheshwari, Osterweil, 1976]. Reduction from 3-SAT.

- Solid edges have uncertainty interval $[0, 1]$, dashed edges have $\{0\}$.
- T^+_{uv} = minimum over w of shortest path P with uv minus shortest path without uv
- Without loss of generality $w_e = 0$ if $e \in P$ and $w_e = 1$ otherwise
- Path is *consistent* if it traverses at most one endpoint of every dashed edge
- $\exists$ Satisfying assignment $\Leftrightarrow$ $\exists$ Consistent path $\Leftrightarrow$ $T^+_{uv} = -1$



NP-hardness for minimum cost perfect matching



Reduction from 3-SAT.

- Variable gadgets $= K_{2,2}$ and
- clause gadgets $= K_{3,3}$
- Edges in gadgets have uncertain edge weights $[0, 1]$, and $\{0\}$ otherwise
- $T^{-}_{uv} = 1 \Leftrightarrow \exists$ Satisfying assignment

Interesting questions

- What aspect of a combinatorial problem allows to compute the thresholds in polynomial time?
- Do the thresholds of inclusion and of exclusion have the same complexity ?

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Thank you for your attention