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# Two complexity results on Spanning-Tree Congestion Problems

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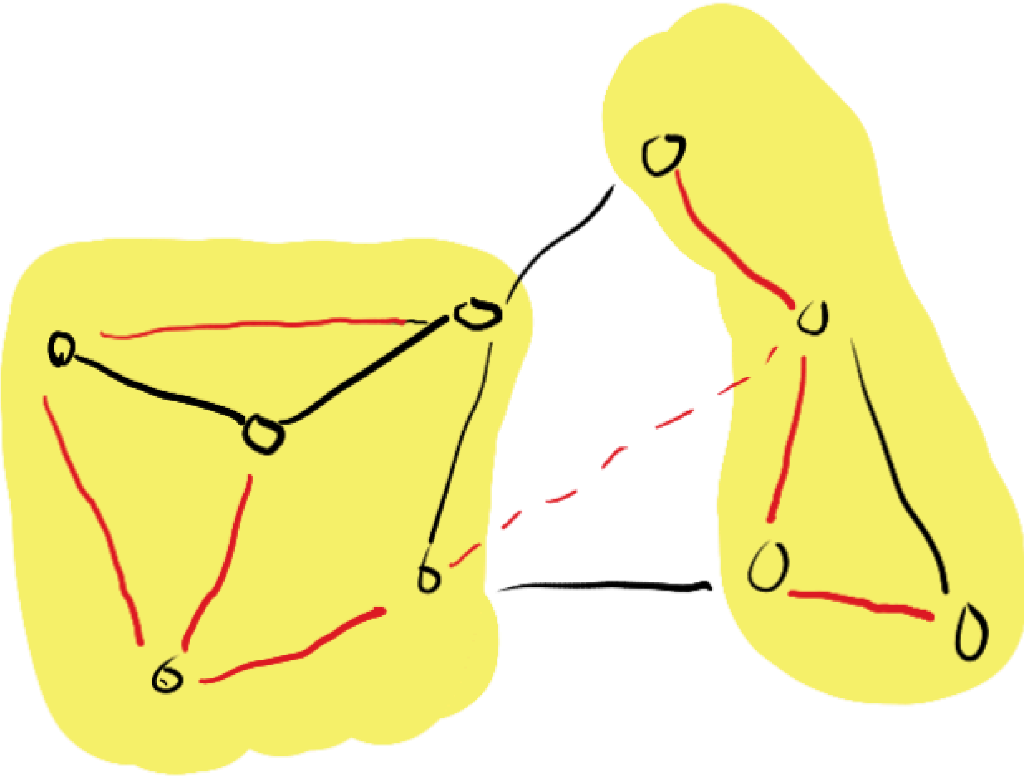
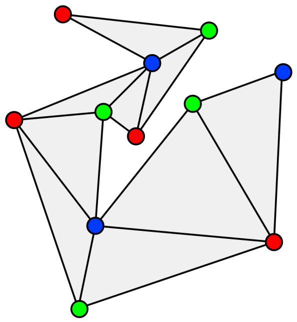
# Spanning Tree Congestion

- graph $G = (V, E)$
- spanning tree $T \subseteq E$
- edge $e \in T$ splits tree, and hence defines a cut
- size of cut = congestion
- congestion of tree = maximum over all tree edges
- find spanning tree with minimal congestion [1984]



# NP-hard for

- general graphs [Löwenstein 2010]
- planar graphs [Otachi ea 2010]
- chain graphs
- split graphs [Okamoto ea 2011]



# polynomial time algorithms for

- outerplanar graphs [Bodlaender ea 2012]
- co-chain graphs [2015]
- interval graphs [Lin, Lin 2025]
- 2 dimensional tori [Kozawa, Otachi 2009]
- 2 dimensional Hamming graphs [Kozawa, Otachi 2011]

# Complexity depending on maximum degree $\Delta$

- NP-hard for $\Delta \geq 8$ [Lampis ea 2025]
- NP-hard for $\Delta \geq 3$ [our work] reduction from M21P1N-SAT

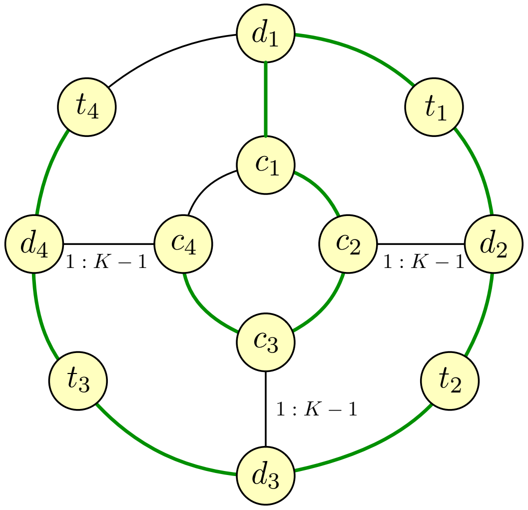



Figure 2: 3-regular gadget simulating higher degree vertices

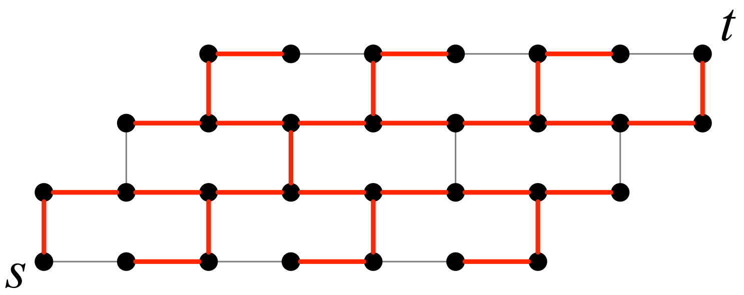



Figure 3: gadget simulating double weight edges. Congestion 1 if not part of spanning tree, and $K-1$ otherwise

# Complexity depending on $K$

is the spanning tree congestion $\leq K$?

- polynomial time solvable for $K = 1, 2, 3$
- NP-hard for $K \geq 10$ [Otachi ea 2010]
- NP-hard for $K \geq 5$ [Lưu, Chrobak 2023]
- open for $K = 4$
- polynomial time solvable for $K$-edge connected graphs [our work]

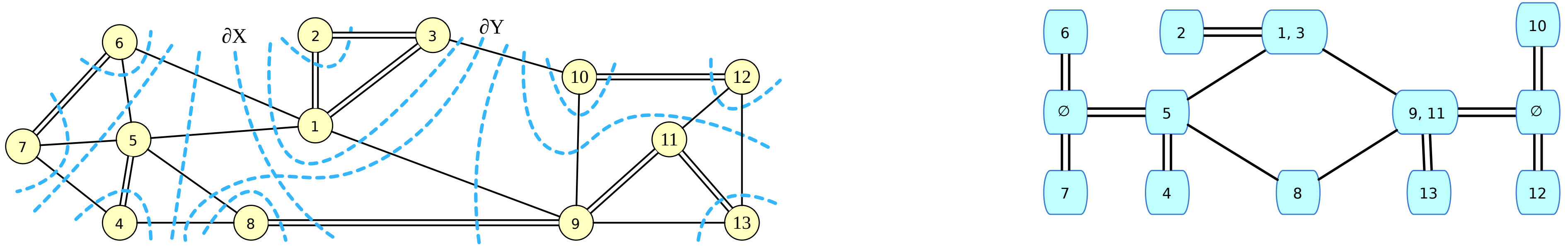



Figure 4: $2$-cuts in the cactus graph representation $C_G$ correspond to $K$-cuts in $G$

# Thank you