Comparison of Failures and Attacks on Random and Scale-free Networks

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Outline

- Context
- Previous results
- New attack strategies

Context

Two kind of disruptions:

- Failures
- Attacks

Evaluation of the damage:
size of connected components

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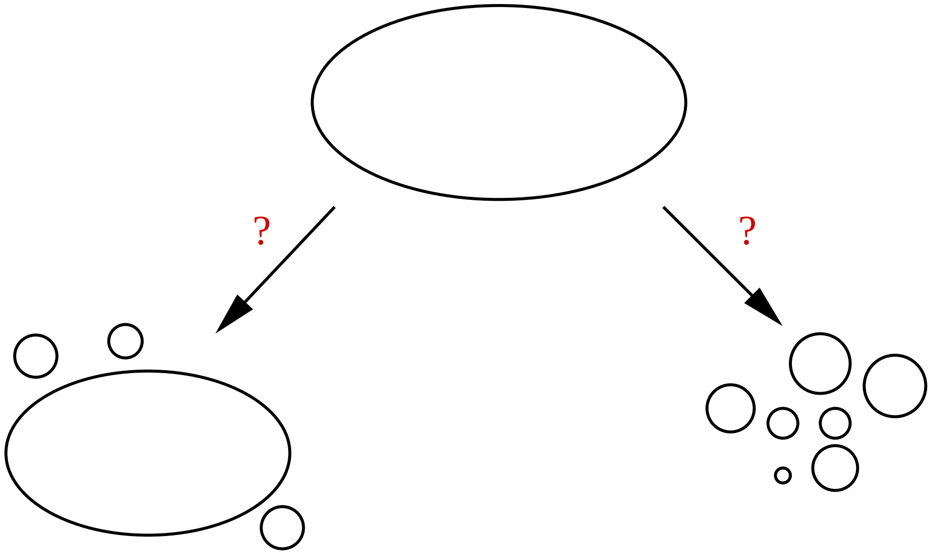

Modeling

- Failures
- Attacks
- Networks

- Failures
  - Random removal of nodes or links
- Attacks
- Networks

Modeling

- Failures
- Attacks
  - Removal of chosen nodes or links
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Modeling

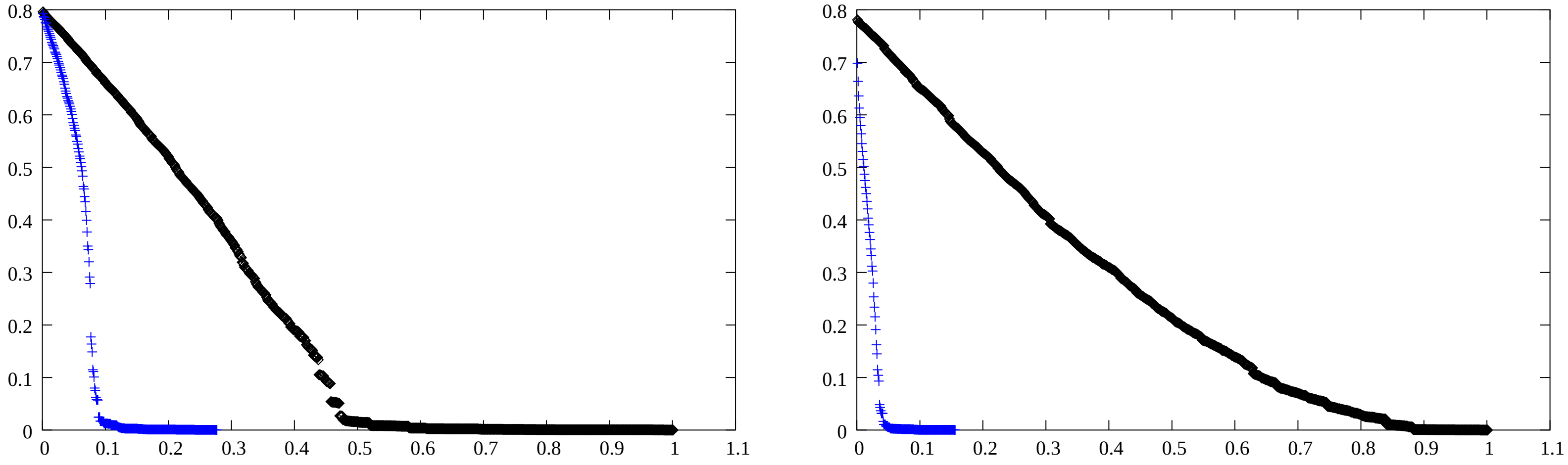
- Failures
- Attacks
- Networks :
  - Random graphs (Erdös et Renyi)
    all nodes have the same importance
  - Scale-free graphs ($p_k \sim k^{-\alpha}$)
    presence of hubs

Experimental results

[Albert, Jeong et Barabási, 2000]

— Node failures

— Attacks (highest degree nodes first)



Random graphs Scale-free graphs

Experimental results

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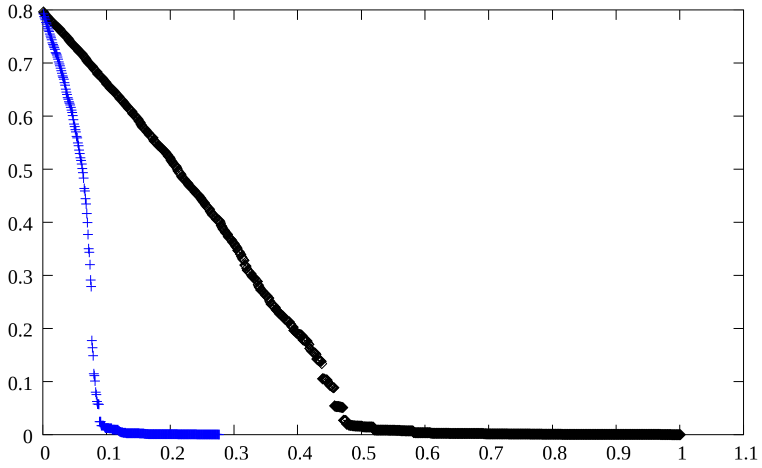
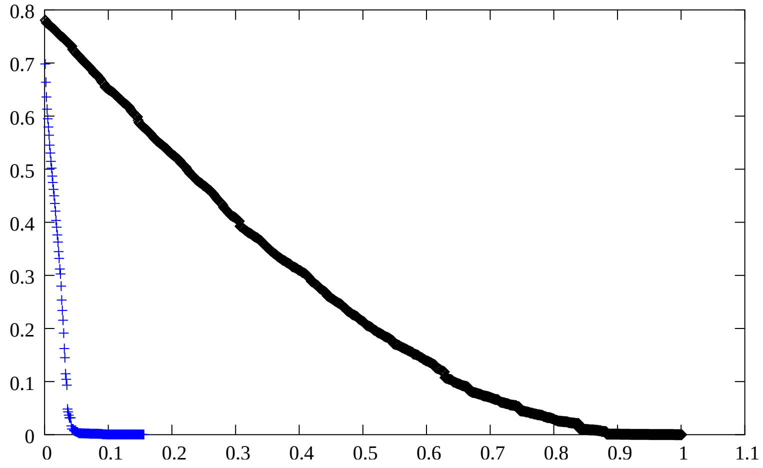


Random graphs



Scale-free graphs

Study of threshold values

Example of theoretical result

[Cohen *et al*, 2000]

Random graph with degree distribution p_k.

Almost surely connected if:

$$\langle k^2 \rangle - 2\langle k \rangle \geq 0$$

$$\sum_{k=0}^{\infty} k(k-2)p_k \geq 0$$

Random removal of a fraction p of nodes:

$$p_k(p) = \sum_{k_0=k}^{\infty} p_{k_0} \binom{k_0}{k} (1-p)^k p^{k_0-k}$$

$$p_c = 1 - \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

Outline of theoretical results

[Callaway *et al*, 2000, Cohen *et al*, 2000, 2001]

- Scale-free graphs
  - No threshold for failures
  - Very low threshold for attacks
- Random graphs
  - Similar behavior for both cases

Failures and attacks on links

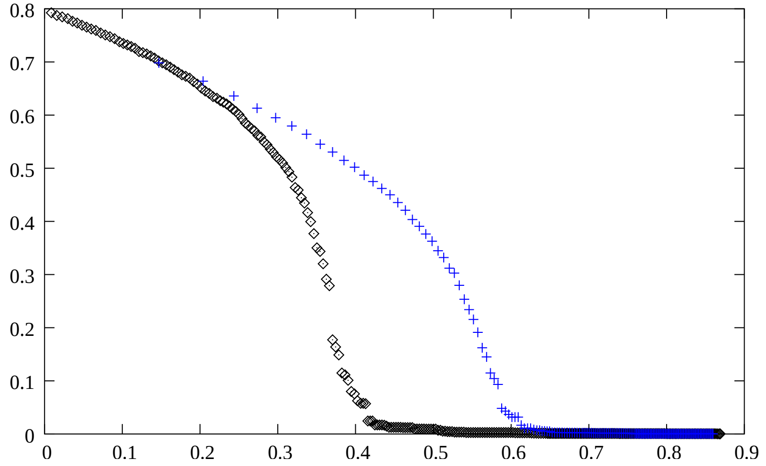
Attacks on nodes $\longrightarrow$ attack strategy for links

Efficiency of attacks due to the large number of links removed?

Failures and attacks on links

Attacks on nodes $\longrightarrow$ attack strategy for links

— Random graph
— Scale-free graph



Attacks on links

Failures and attacks on links

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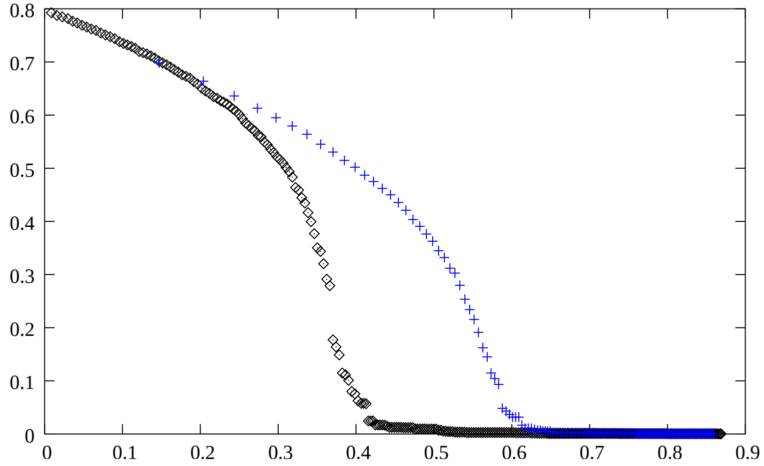

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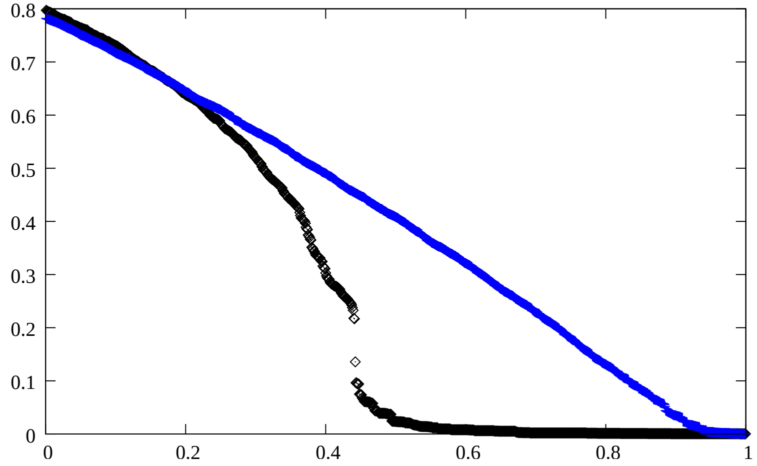


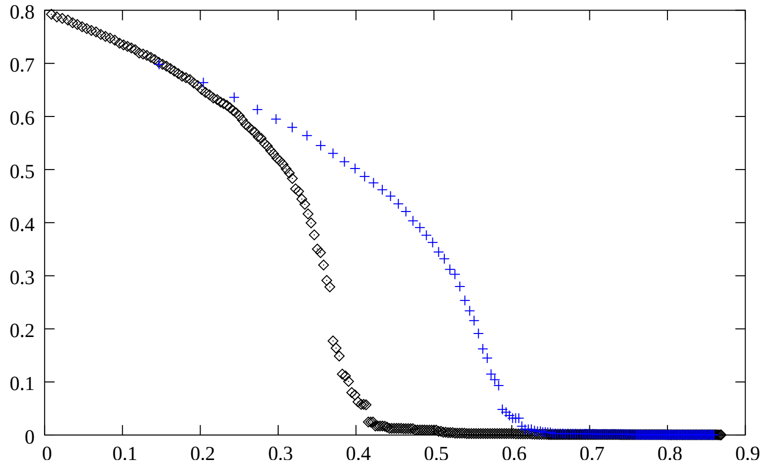
Link failures

Failures and attacks on links

Attacks on nodes $\longrightarrow$ attack strategy for links

— Random graph

— Scale-free graph



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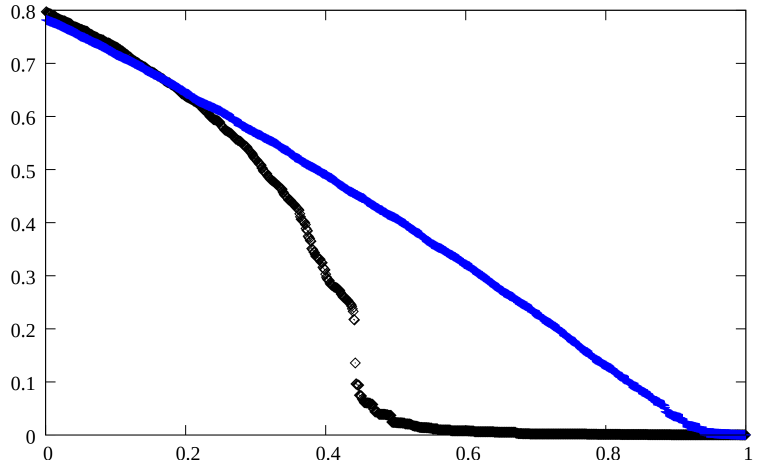

Link failures

Scale-free graphs : no threshold for link failures

New attack strategies

[Molloy et Reed, 1995]

A graph is almost surely connected if

$$\langle k^2 \rangle - 2\langle k \rangle \geq 0$$

$$\sum_{k=0}^{\infty} k(k-2)p_k \geq 0$$

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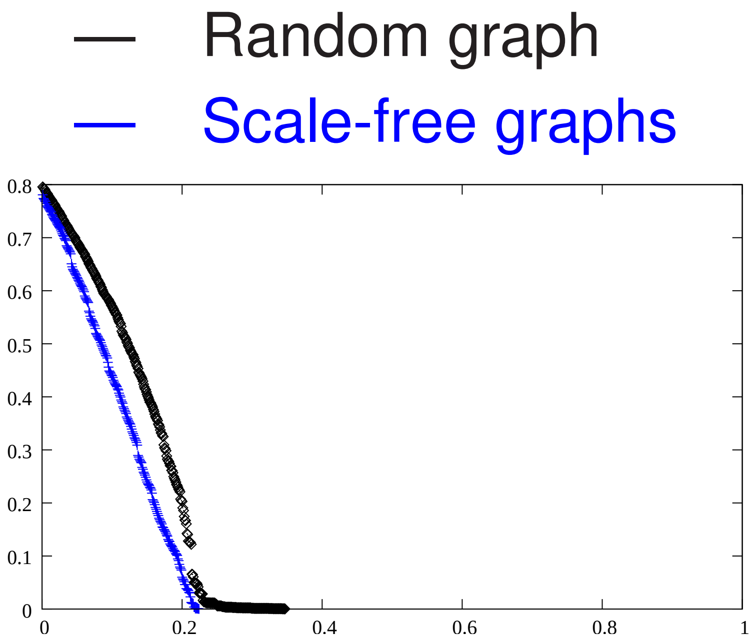
$\longrightarrow$ new attack strategies.

Nodes

Strategy : Random removal of nodes of degree ≥ 2



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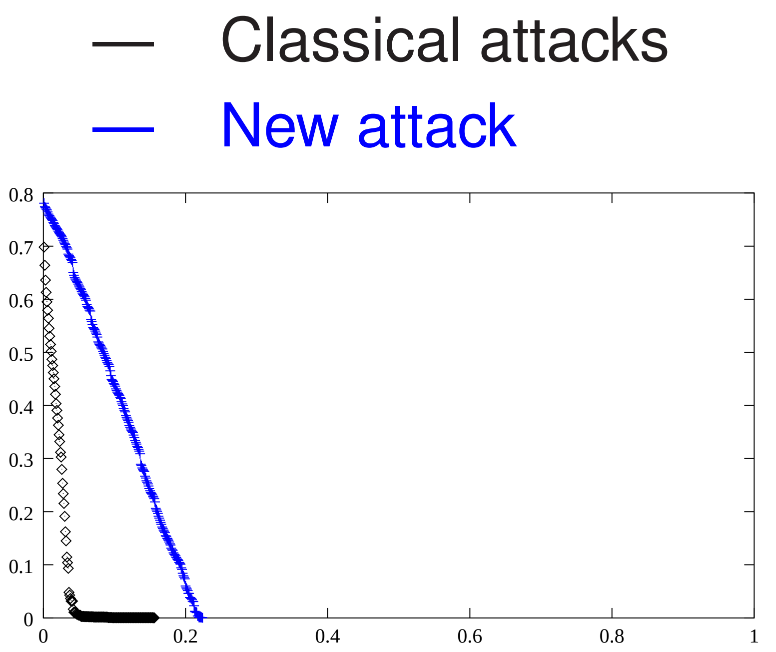

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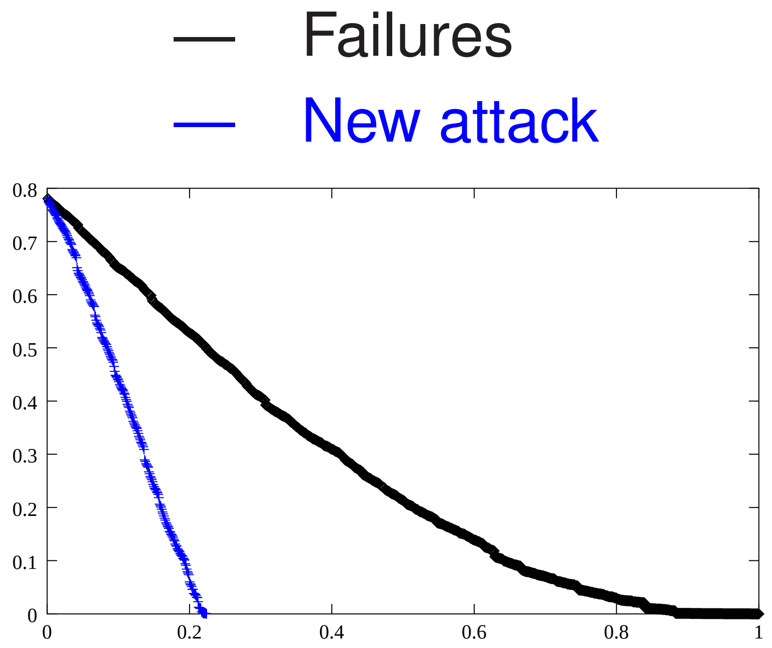

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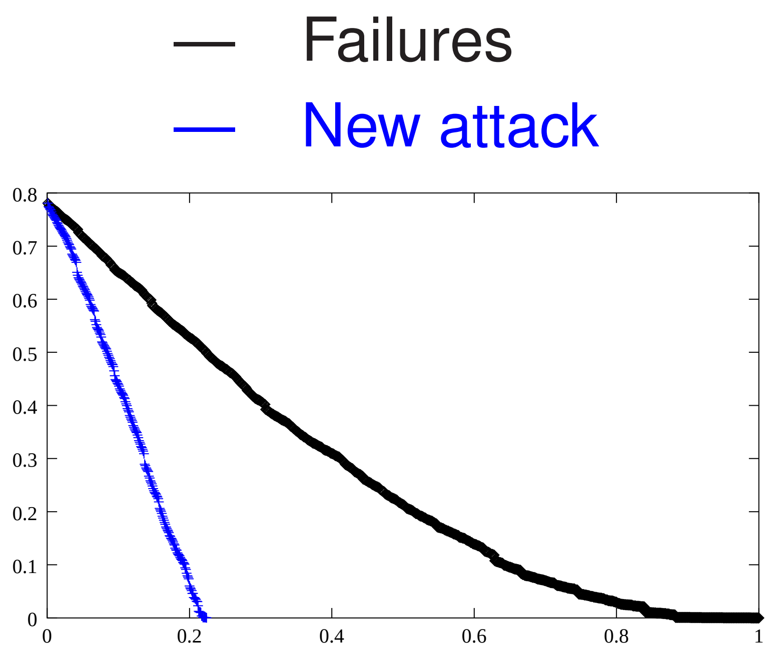


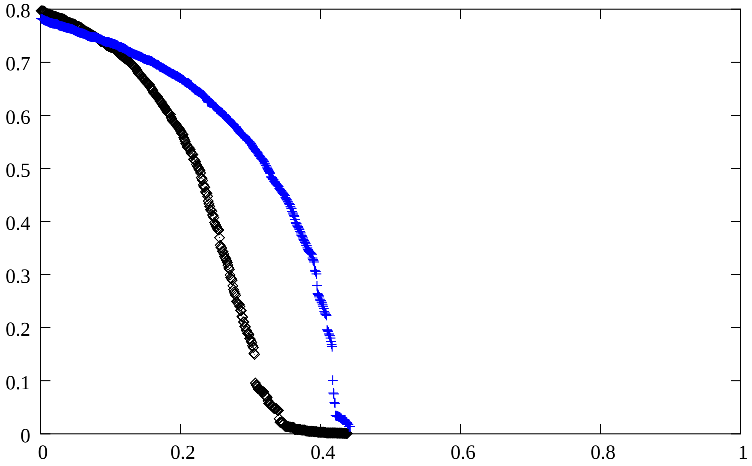
Upper bound for threshold : $1 - p_1$
($\simeq 0.35$ for a scale-free graph with $\alpha = 2.5$)

Strategy :

Remove randomly links between nodes with degree ≥ 2.

— Random graph

— Scale-free graph



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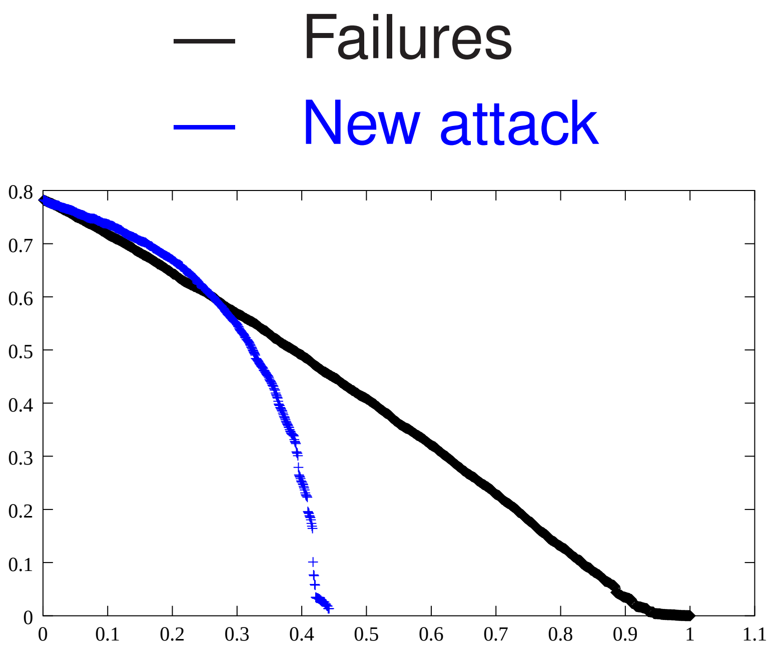

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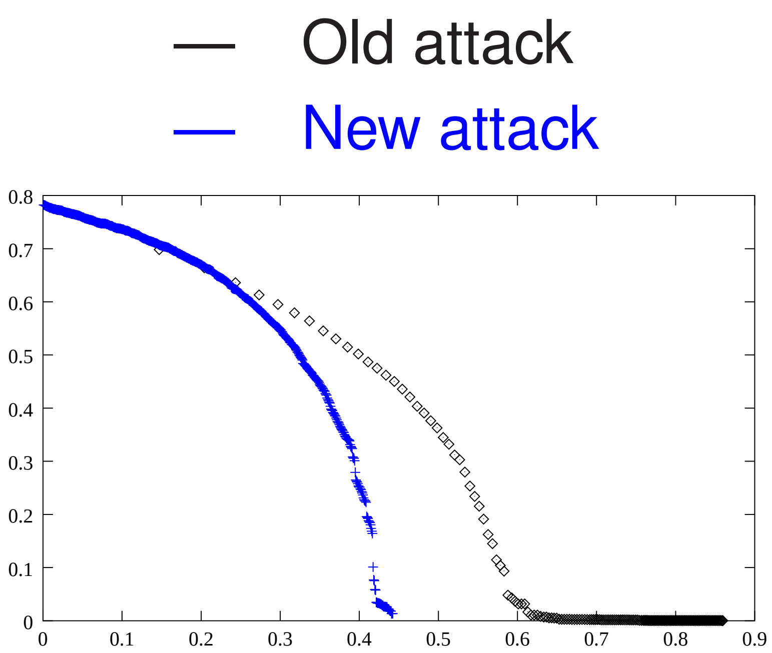


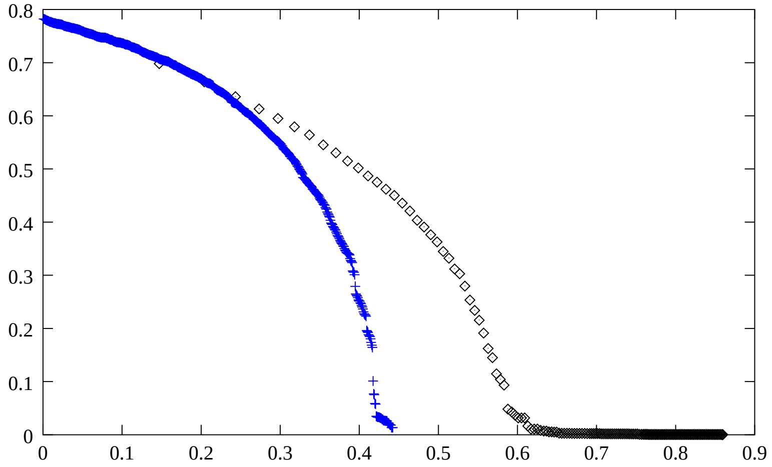
Links

Strategy :

Remove randomly links between nodes with degree ≥ 2.

— Old attack

— New attack



Upper bound for the threshold: $\simeq 0.6$ for scale-free graphs with $\alpha = 2.5$.

Conclusion and perspectives

- Two new attack strategies
- Formal proof for threshold
- New attacks on links more efficient

- Clustering