Analysis of growing graphs as a way to identify their fundamental properties

Work in collaboration with Matthieu Latapy

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Outline

Outline

Typical examples

- The Web graph [KRRT99, AJB99]
  Web pages linked by hyperlinks
  Data research, . . .
  Exploration: Crawl

Typical examples

- The Web graph [KRRT99, AJB99]
- Peer-to-Peer exchanges [Le Fessant 2004, GLS2004]
  Users linked by the exchange of data
  Design of efficient architectures, . . .
  Exploration: Capture from a server

Typical examples

- The Web graph [KRRT99, AJB99]
- Peer-to-Peer exchanges [Le Fessant 2004, GLS2004]
- Internet topology [FFF99, AB99]
  Routers linked by cables
  Protocol design, . . .
  Exploration: `traceroute`

Typical examples

- The Web graph [KRRT99, AJB99]
- Peer-to-Peer exchanges [Le Fessant 2004, GLS2004]
- Internet topology [FFF99, AB99]
- IP exchanges [Soule *et al*, 2005]
  Computers linked by the exchange of IP packets
  Usage analysis, . . .
  Exploration: Capture from a router

Typical examples

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- IP exchanges [Soule *et al*, 2005]

Difficulty of capture:

- Access to the data
- Various technical constraints
- Size

The data set

Four data sets coming from different contexts

- Peer-to-Peer exchanges: e-Donkey trace of 16 hours (P2P)
  - 835,000 nodes
  - 2,800,000 links
- Web: crawl launched from French labs for 4 hours (Web)
  - 700,000 nodes
  - 1,900,000 links
- Internet: traceroutes from one source to 60,000 destinations (Inet)
  - 114,000 nodes
  - 118,000 links
- IP exchanges: 1 week traffic in a large French lab (IP)
  - 470,000 nodes
  - 1,700,000 links

Current situation

Get as much data as possible
(not possible to get all the data)

Consider it as representative of the whole

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Non trivial properties in common

Usual assumptions

- Average degree
  Degree of a node: its number of links

Usual assumptions

- Average degree constant

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- Density
  Number of links / number of possible links

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- Degree distribution
  p_k: number of nodes with degree k

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- Average distance and diameter
  - Distance between two nodes: Minimal number of links to go from one node to the other
  - Diameter: Largest distance in the graph

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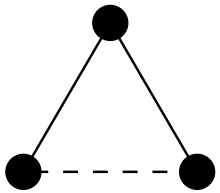

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Methodology

Coarse-grained graphs:

$\longrightarrow$ divide the sample in slices (time, size, ...)

Observe the properties of the sample at the end of these slices

Look for stable properties

$\longrightarrow$ check the usual assumptions

Challenges:

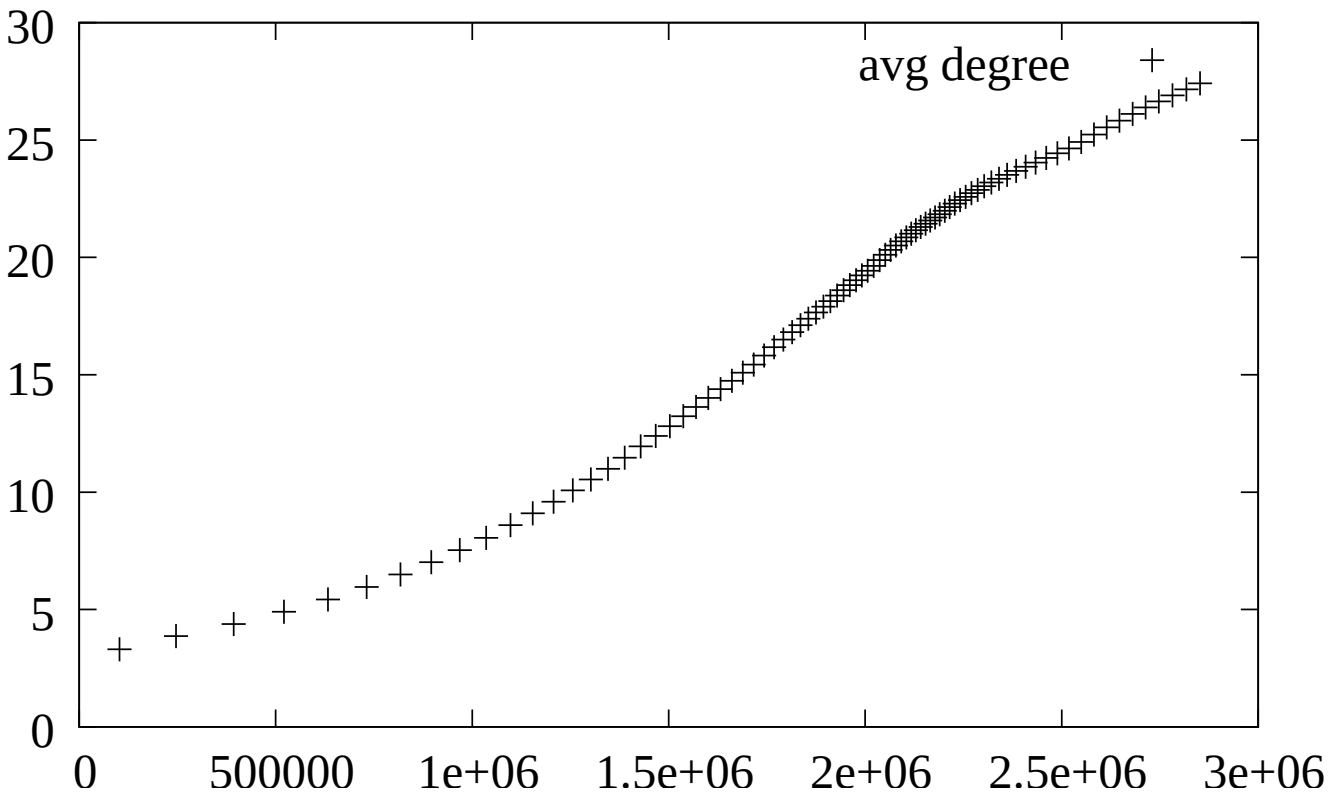
- Data acquisition, dynamics
- Size of data: use of heuristics

Outline

Average degree

Average degree: $2M/N$

Usual assumption: constant



P2P

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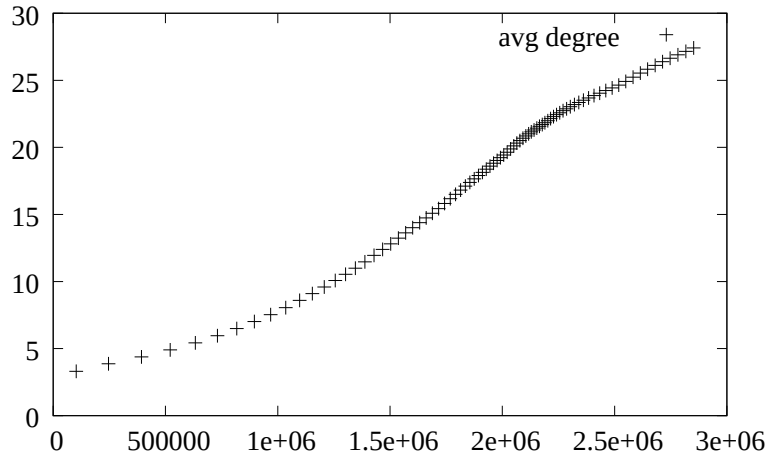
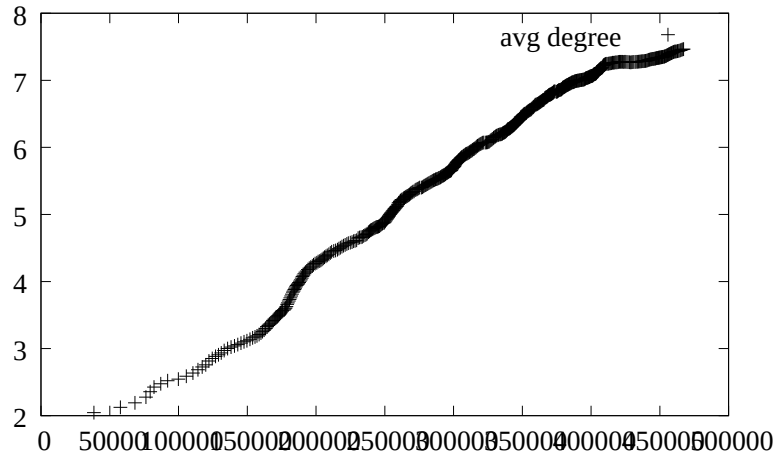
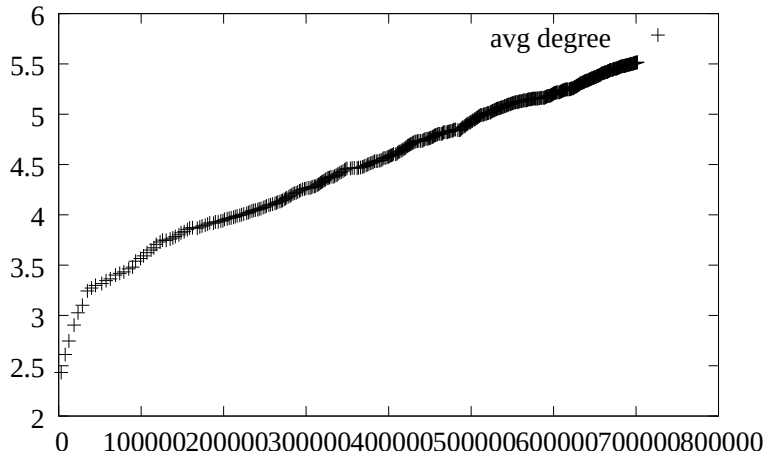
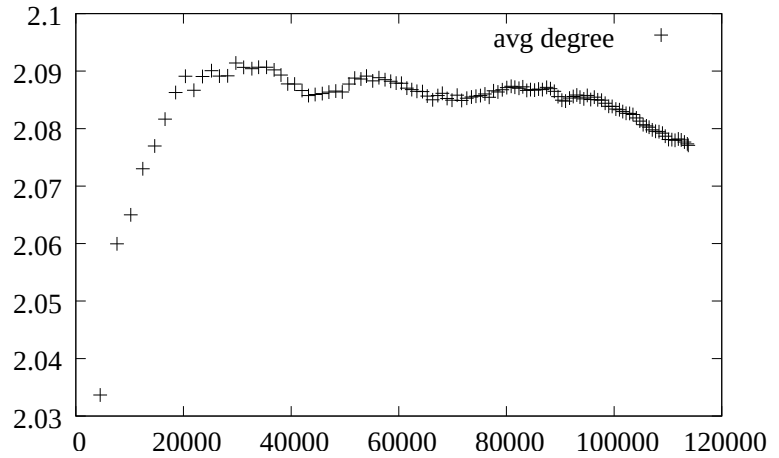




IP

Web

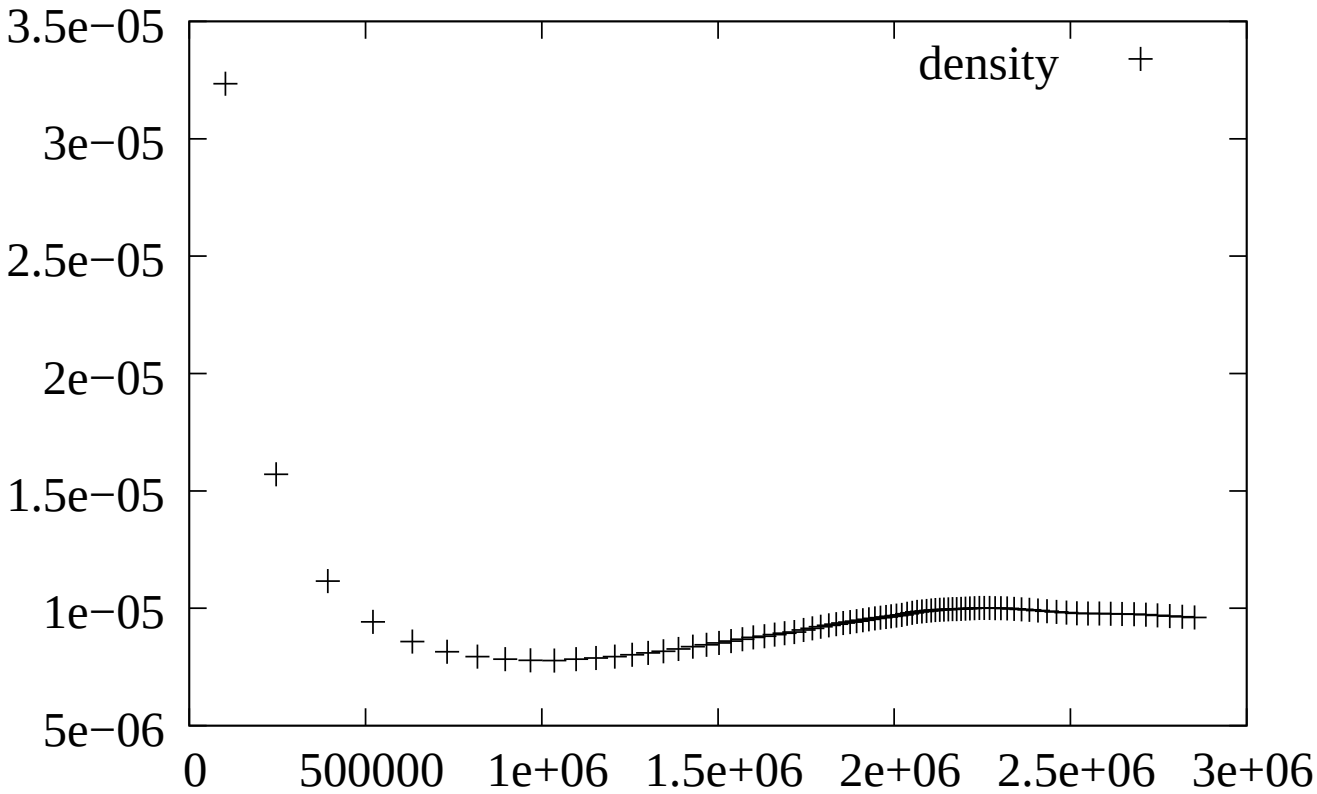




Inet

Density

Density: $2M/N(N-1)$
Usual: tends to 0



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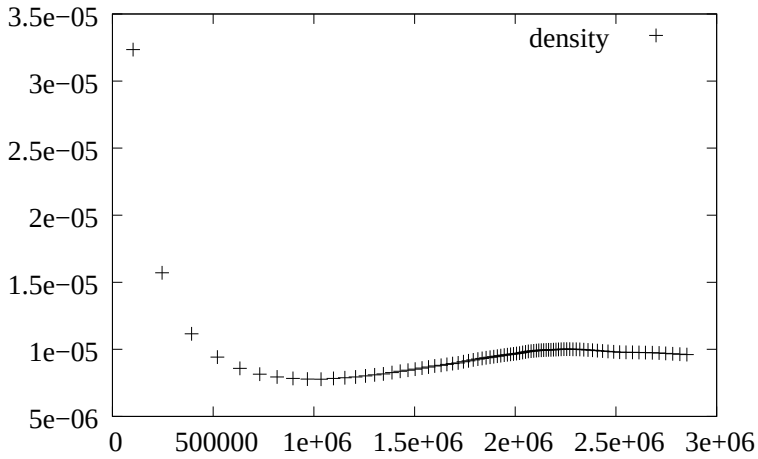

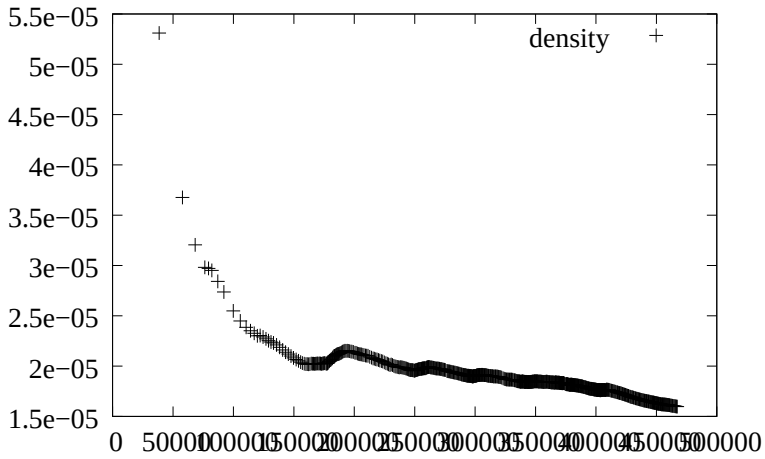

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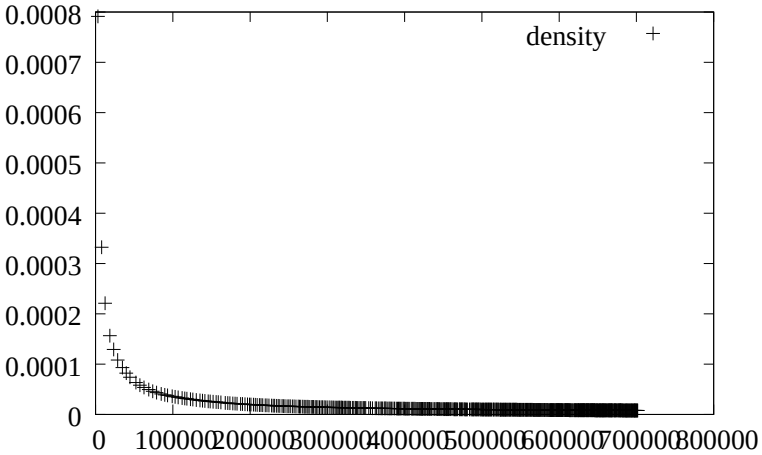

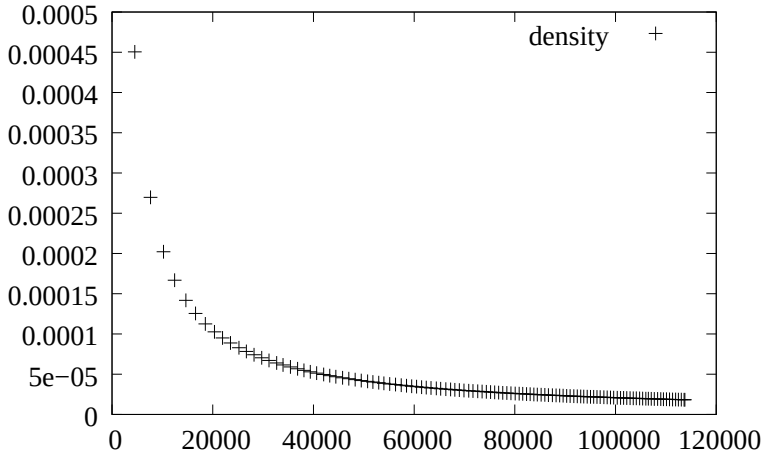

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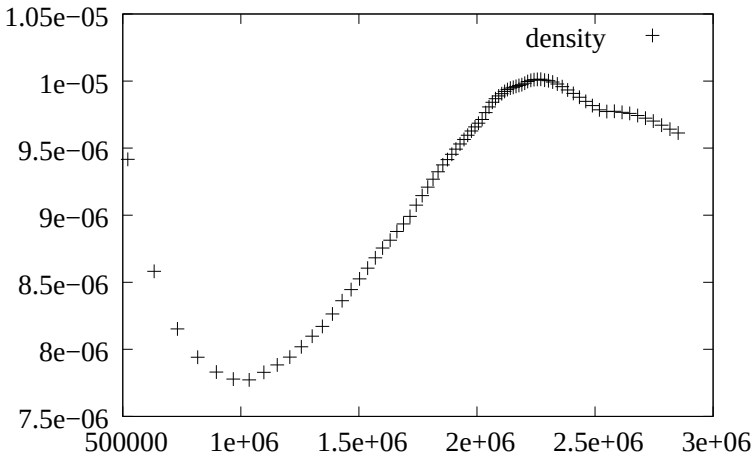

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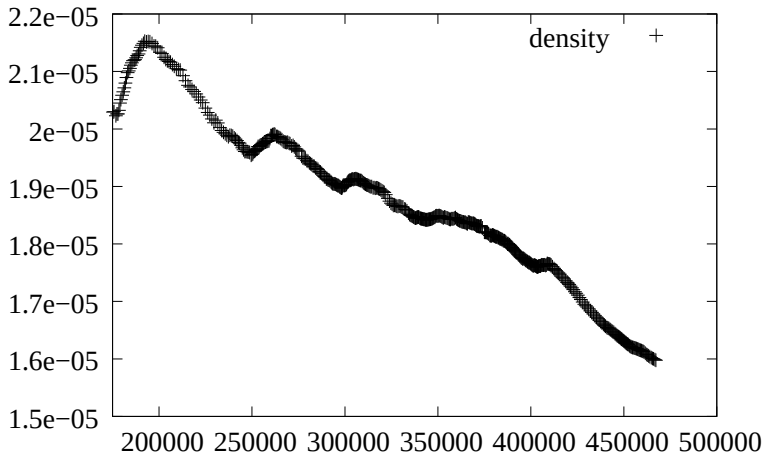

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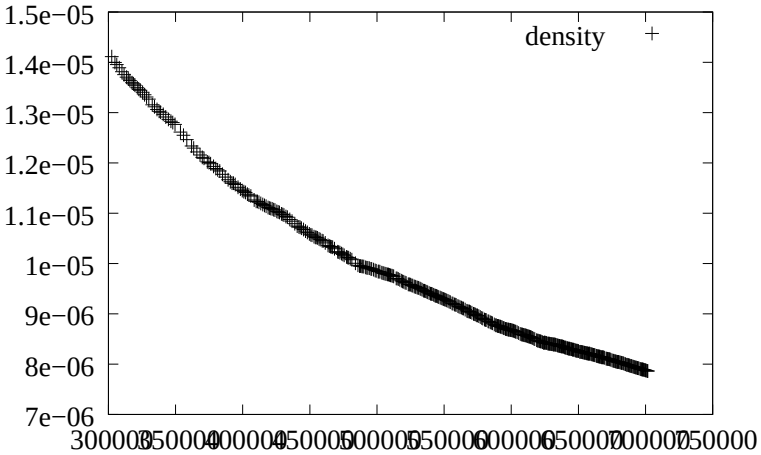

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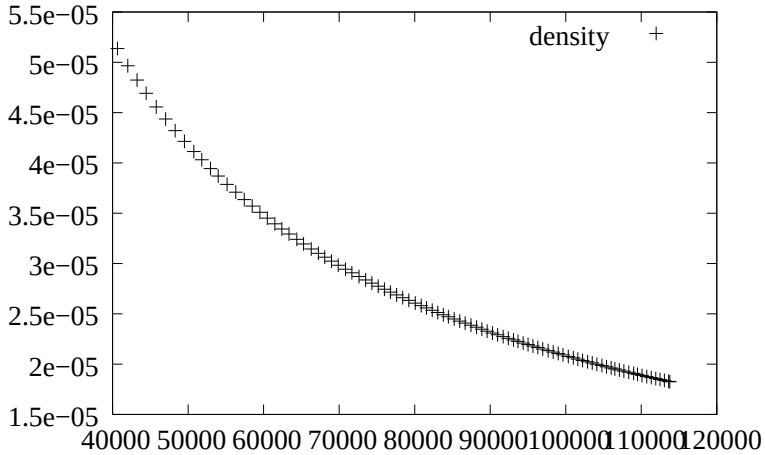
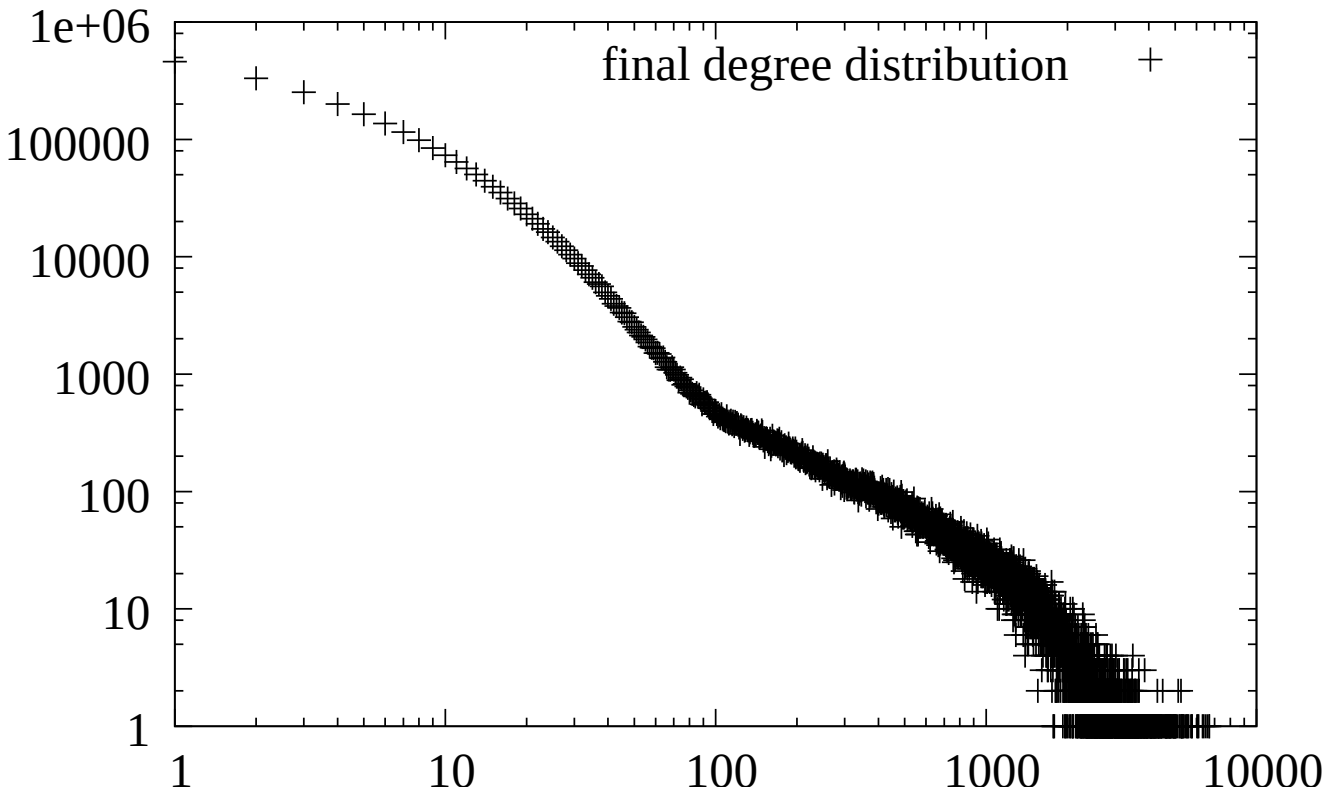


Degree distribution

p_k: number of nodes of nodes with degree k.
Usual assumption: heterogeneous, stable



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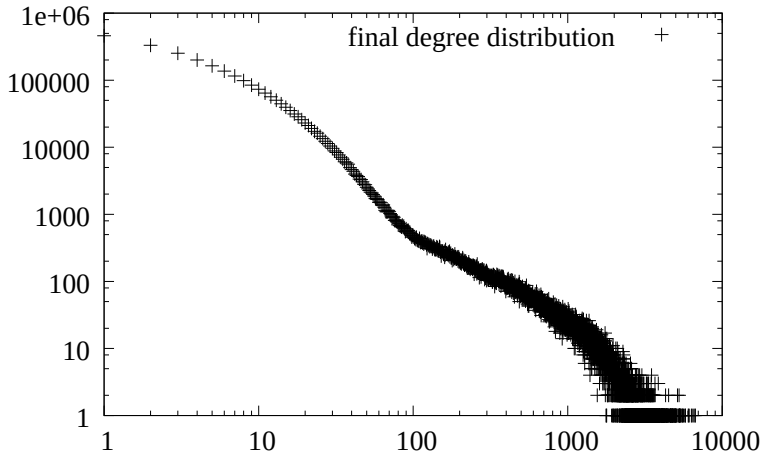

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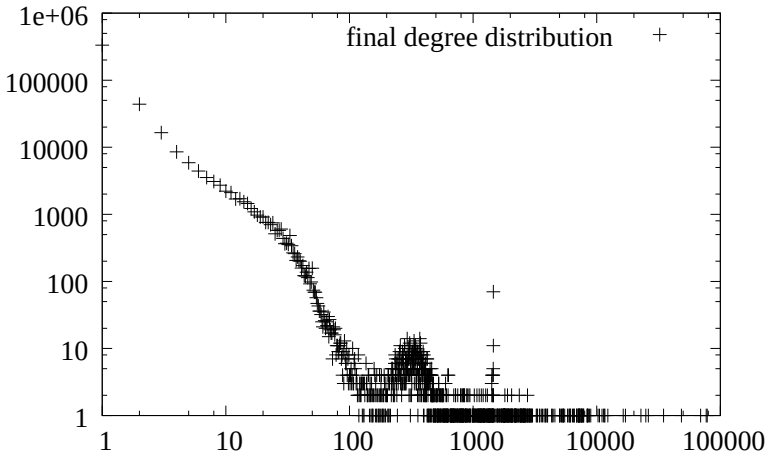

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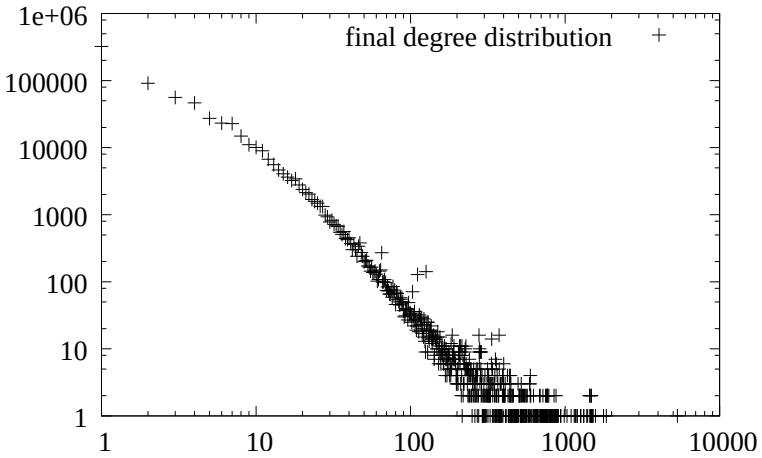

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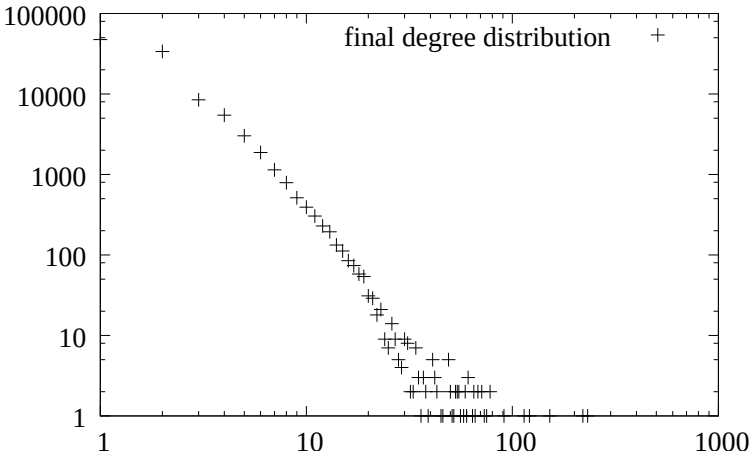
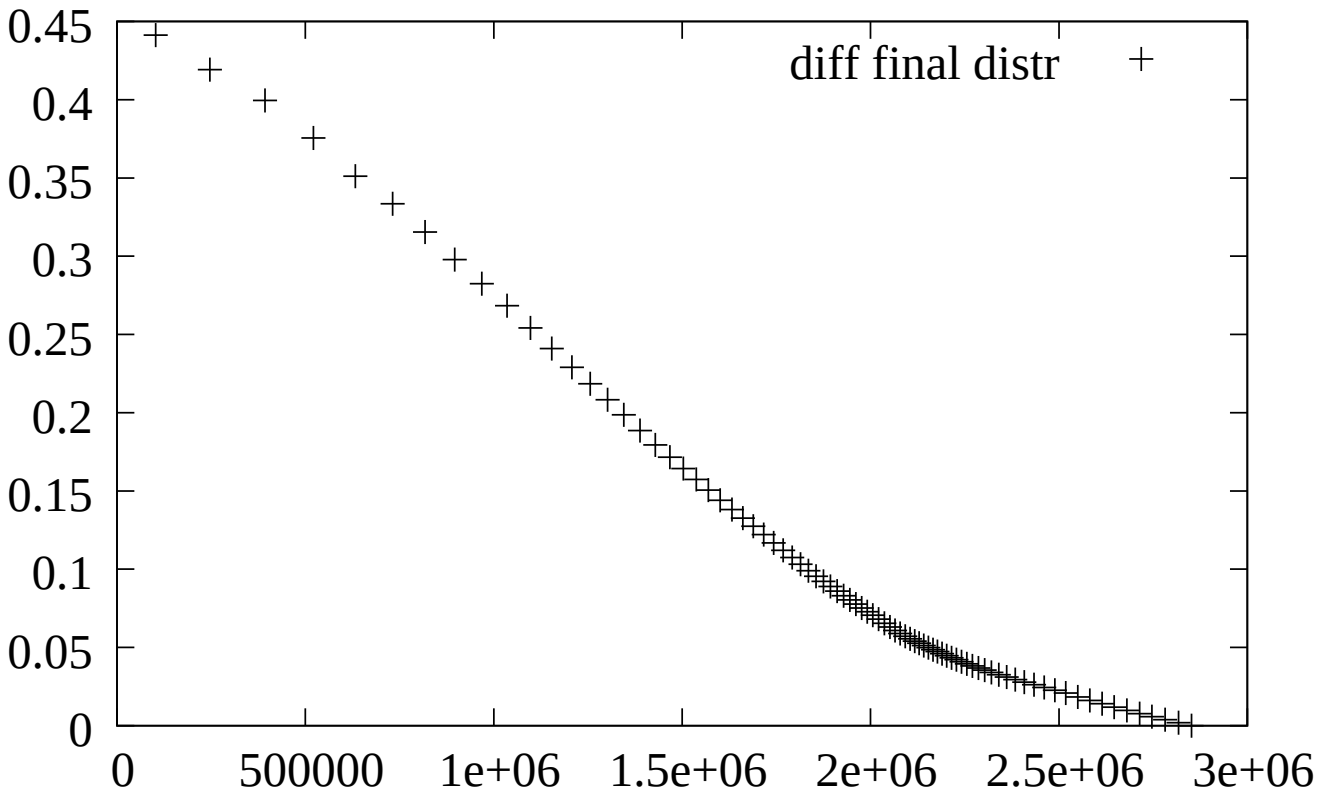


Always heterogeneous

Evolution of the degree distribution

K-S test: maximum difference between cumulative distributions
Difference with last degree distribution



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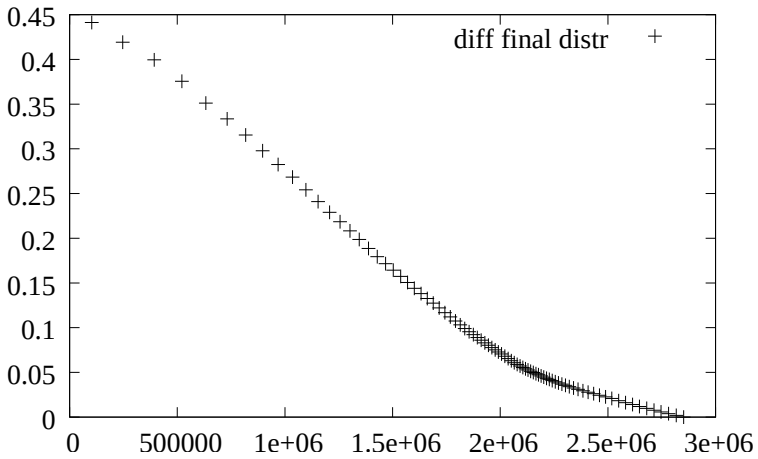

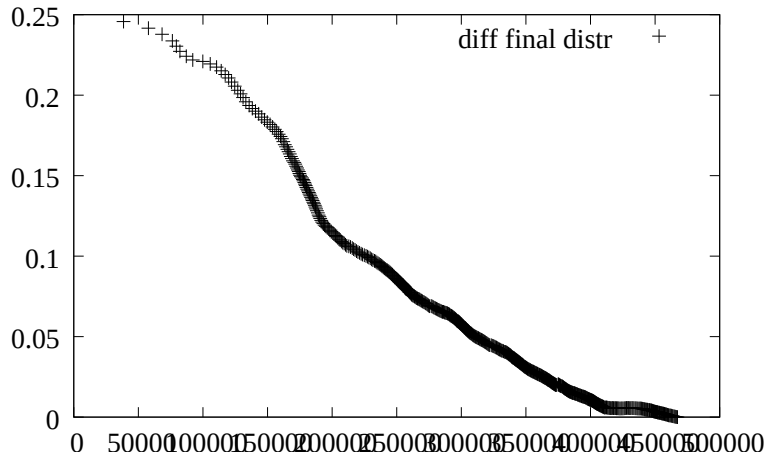

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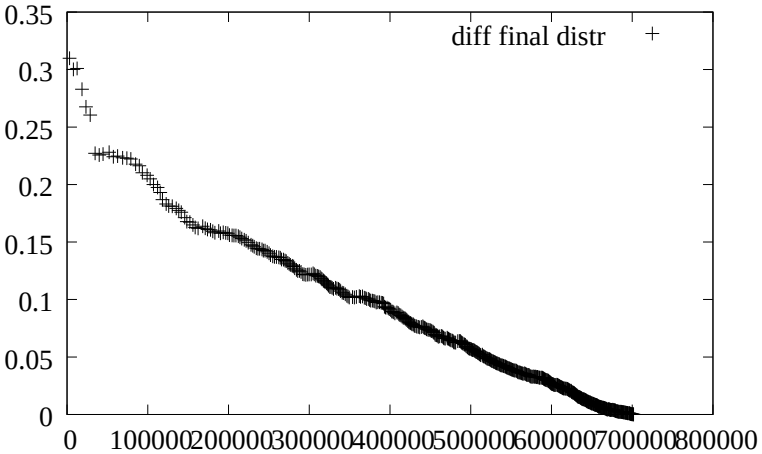
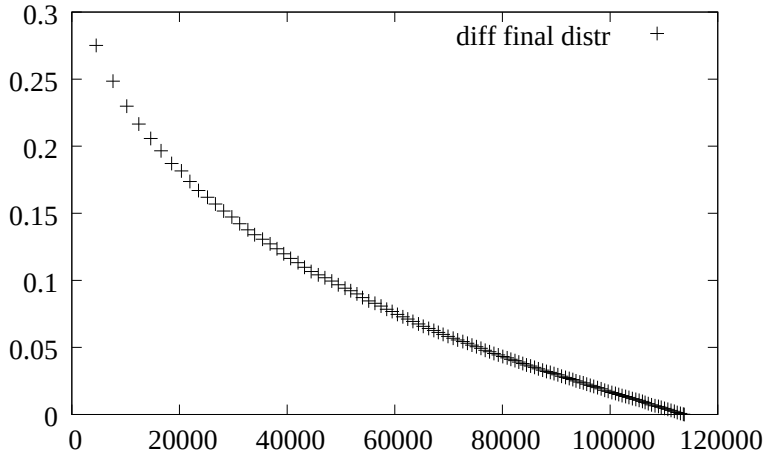
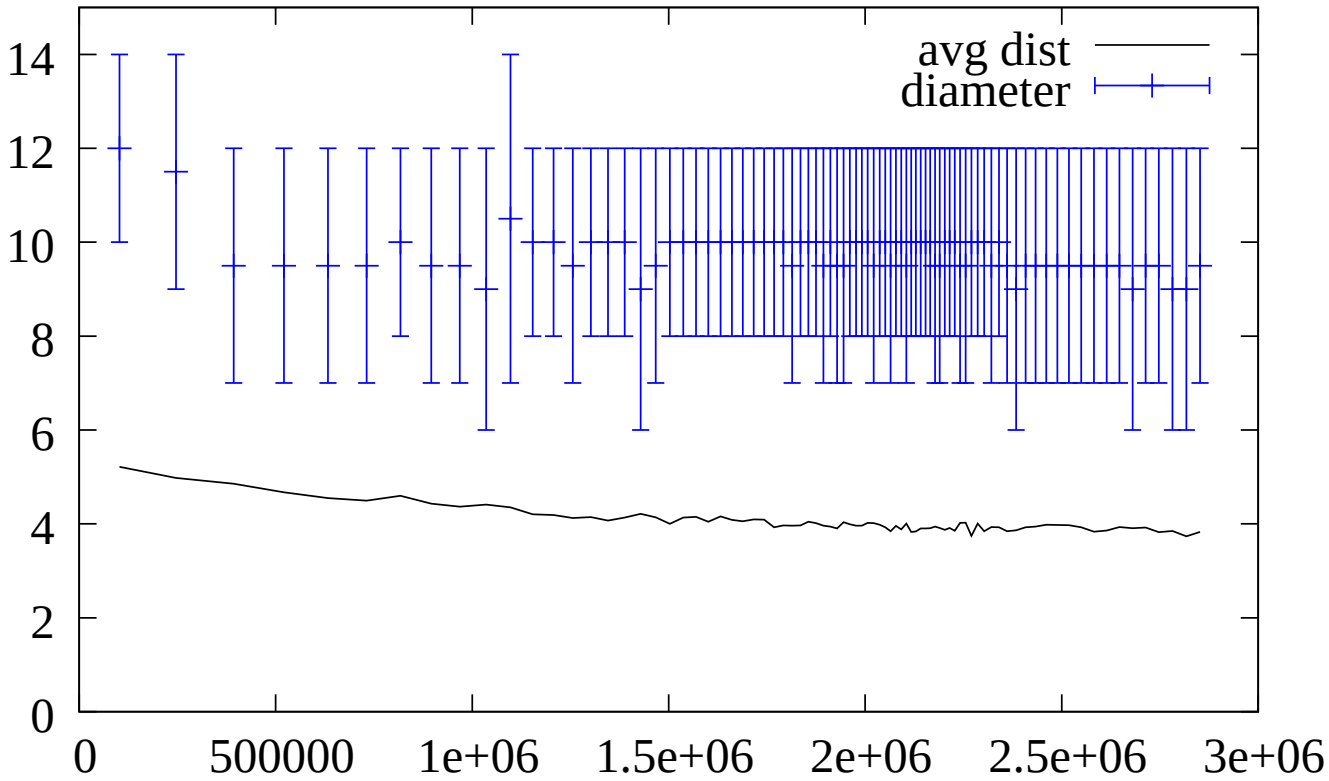




Inet

Average distance and diameter

Average distance: average on all pairs of nodes
Diameter: maximum distance
Use of heuristics
Usual assumptions: small, increasing



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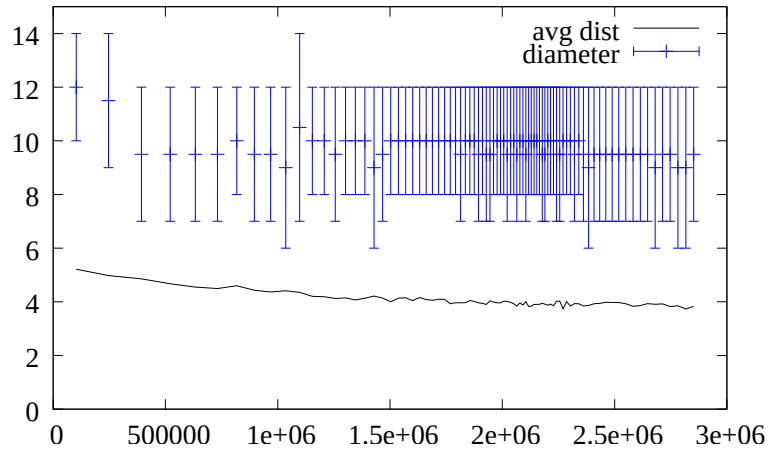

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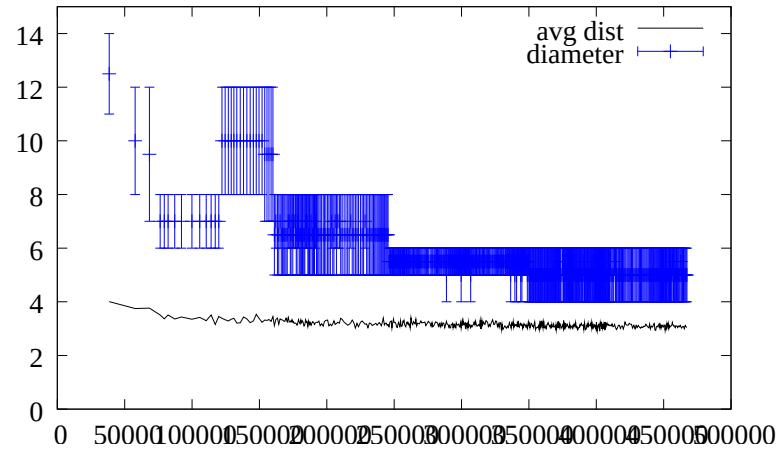

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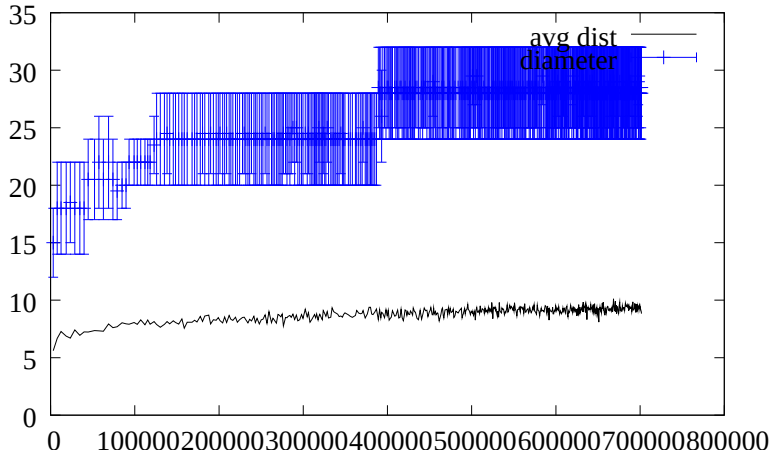

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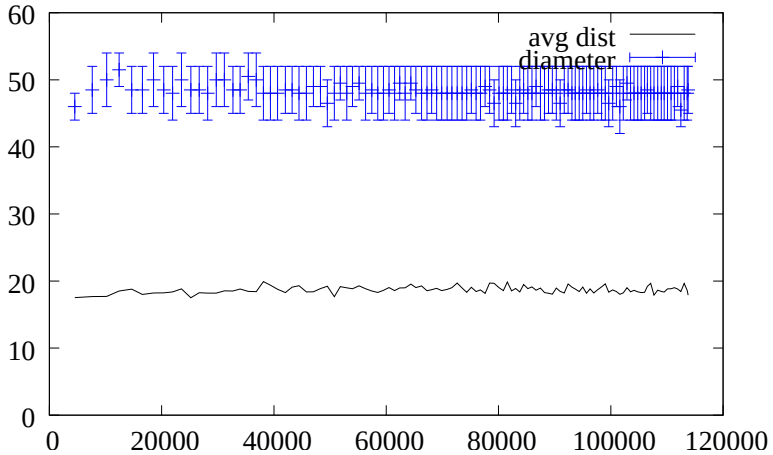

Clustering coefficient

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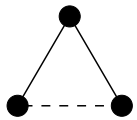
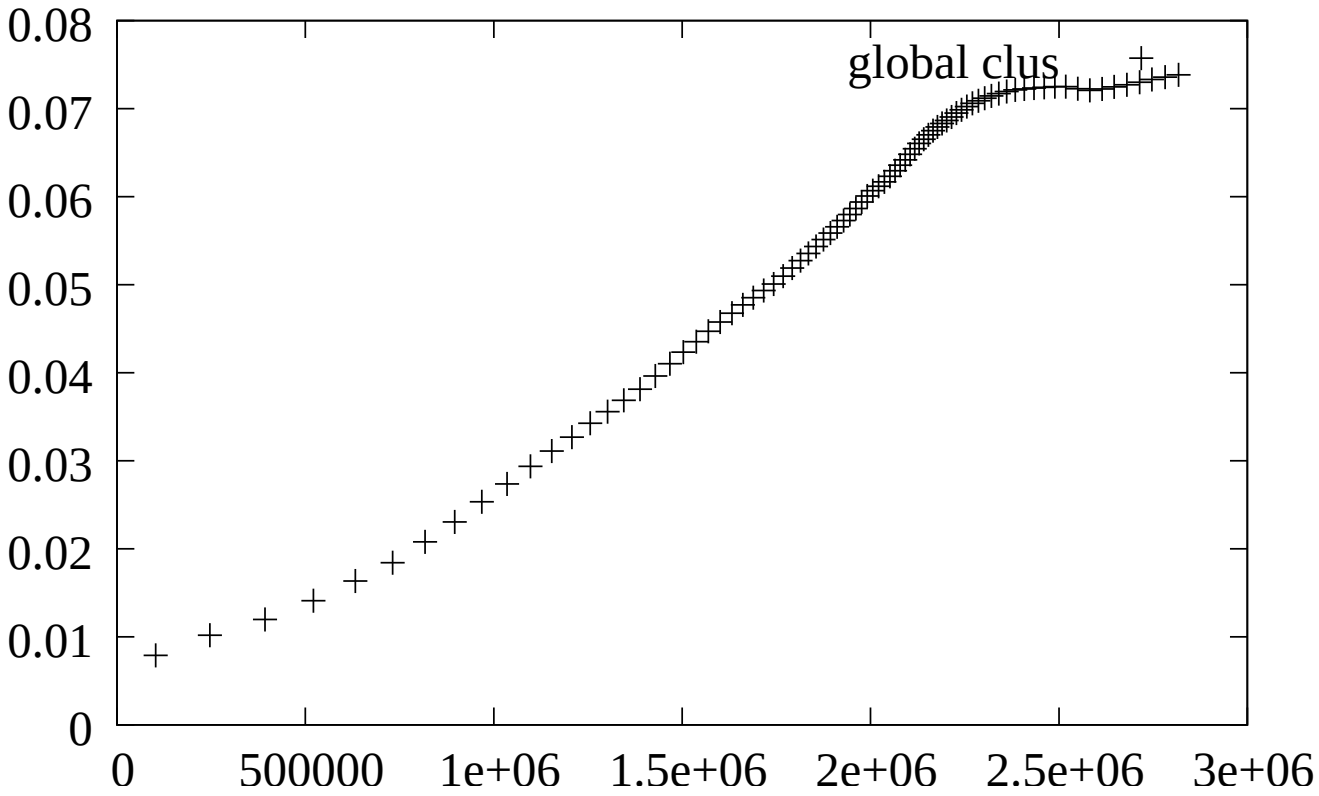


Usual assumption: significantly larger than density, constant



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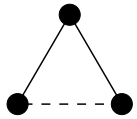

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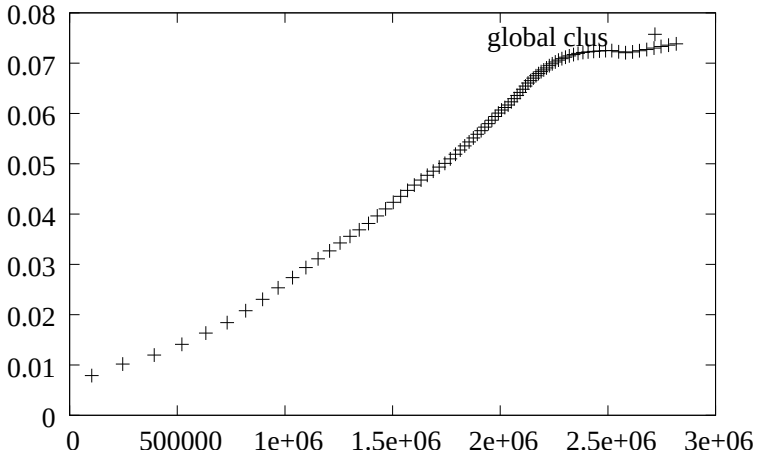

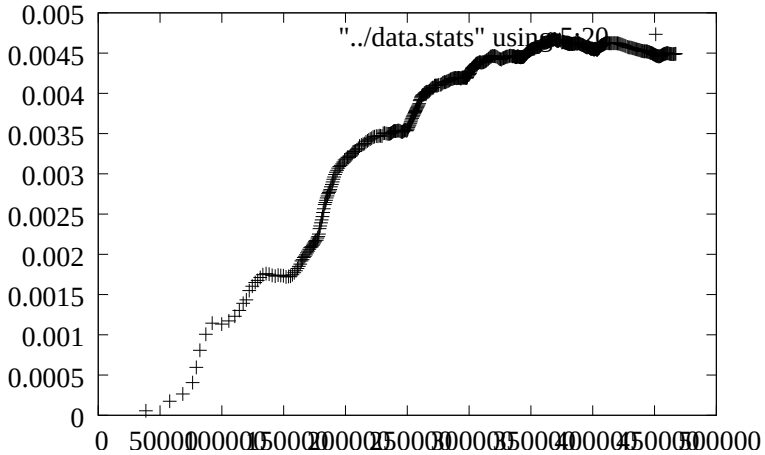

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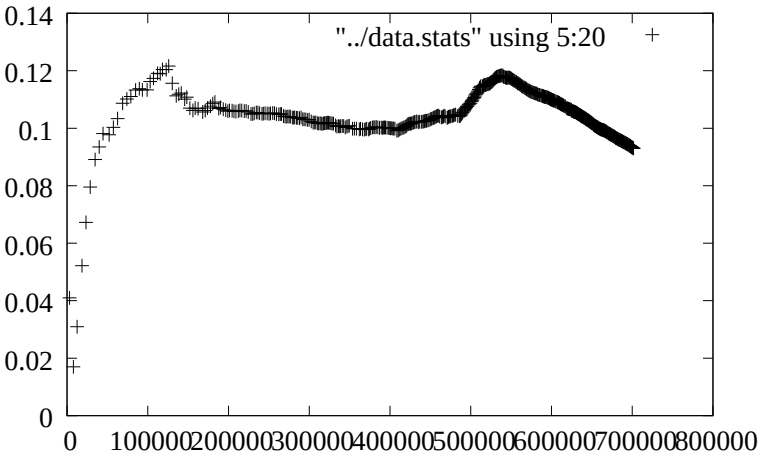

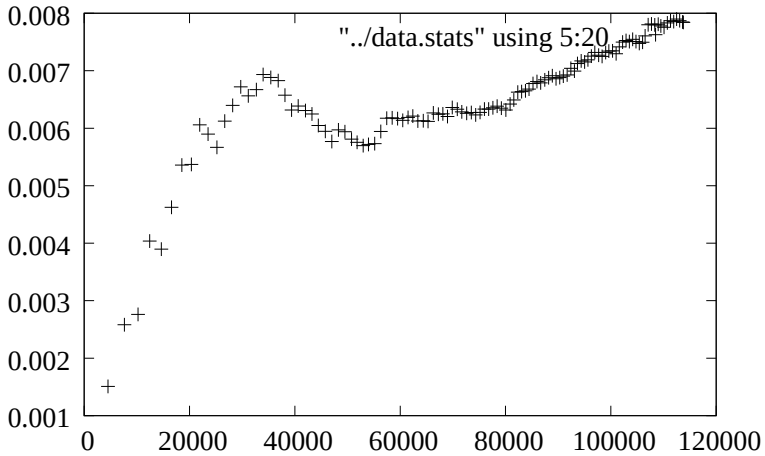

Inet

Conclusion and perspectives

Identification of universal properties

Some classical assumptions validated
Other properties infirmed

Perspectives:

- Study of different/larger data
- Links between these propertie
- New questions for modeling issues
- Investigations on slices of the samples