Efficient Convex Zone Merging in Parametric Timed Automata

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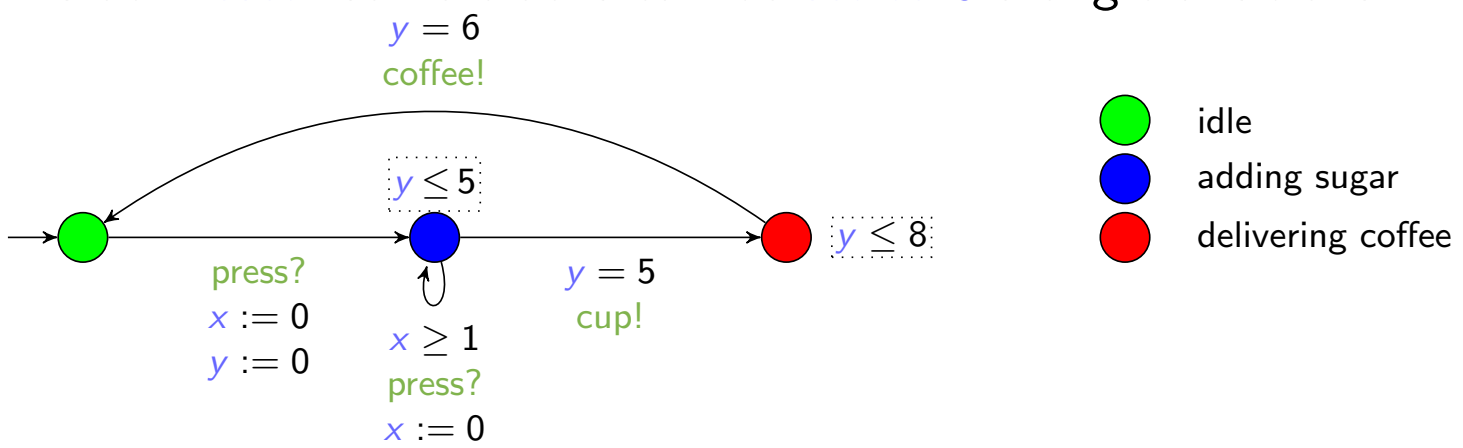
Timed Automaton (TA)

- Finite state automaton (sets of locations and actions) augmented with
  - a set X of clocks [AD94]
    - Real-valued variables evolving linearly at the same rate
    - Can be compared to integer constants in invariants and guards

- Features
  - Location invariant: property to be verified to stay at a location
  - Transition guard: property to be verified to enable a transition
  - Clock reset: some clocks can be set to 0 along transitions



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Parametric Timed Automaton (PTA)

- Finite state automaton (sets of locations and actions) augmented with
  - a set X of clocks [AD94]
    - Real-valued variables evolving linearly at the same rate
    - Can be compared to integer constants in invariants and guards
  - a set P of parameters (unknown constants) [AHV93]

- Features
  - Location invariant: property to be verified to stay at a location
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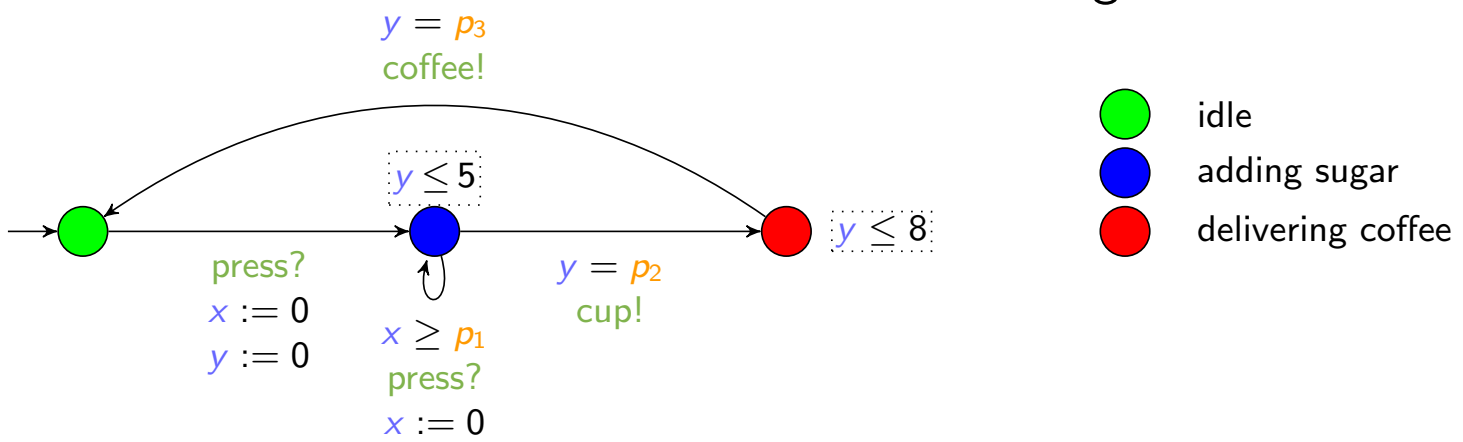

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Parametric Zone Graph (PZG)

- Symbolic state: a pair with a location and an attached parametric zone (constraint)
- Parametric zone: a set of valuations defined by conjunctions of constraints on clocks and parameters

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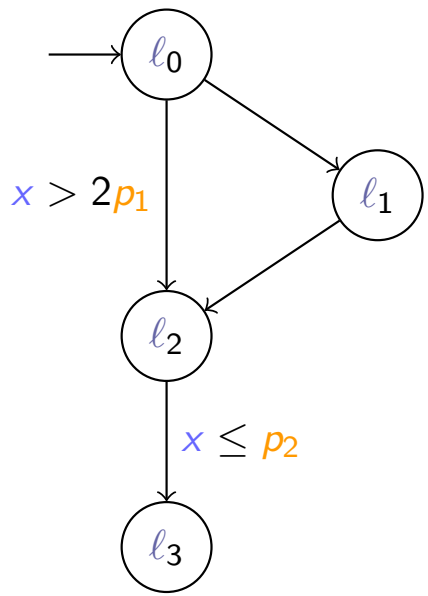

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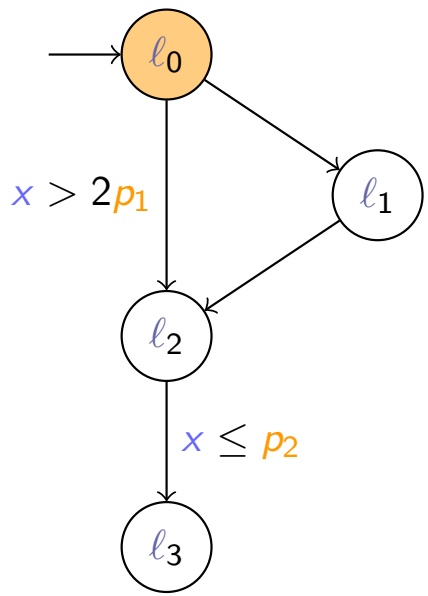

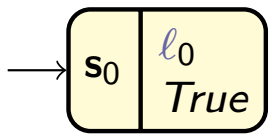

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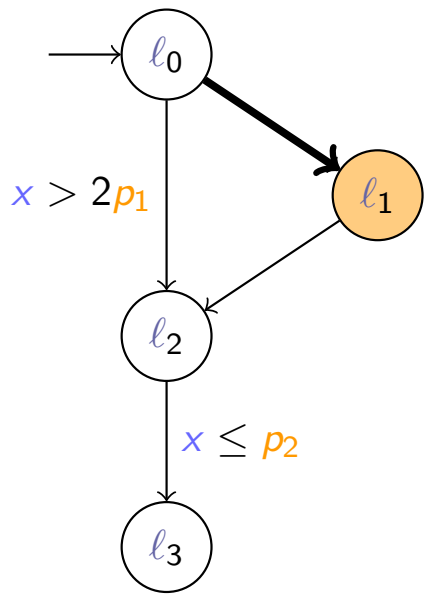

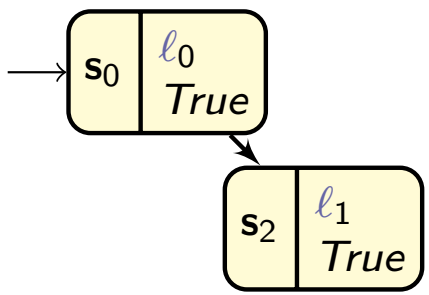

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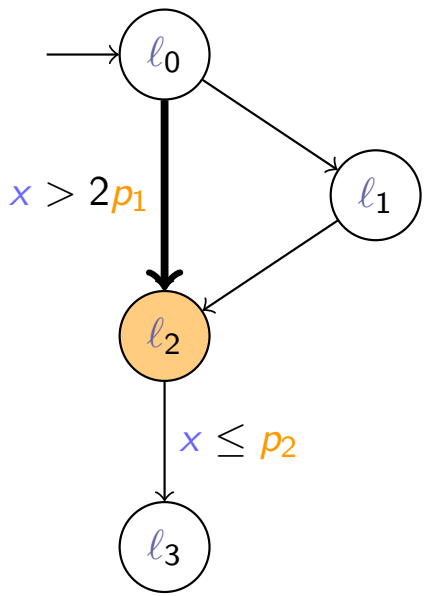

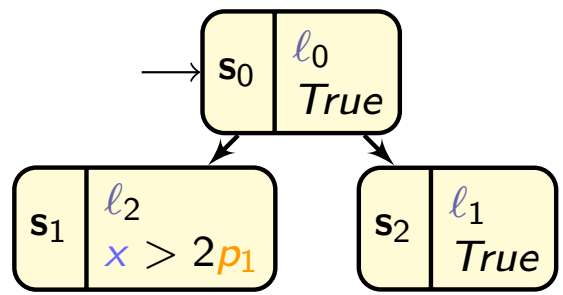

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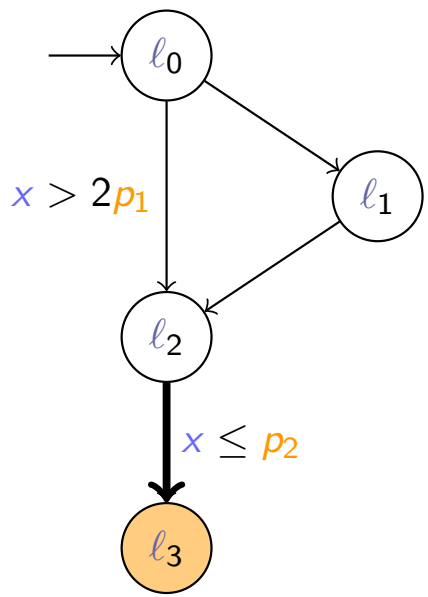

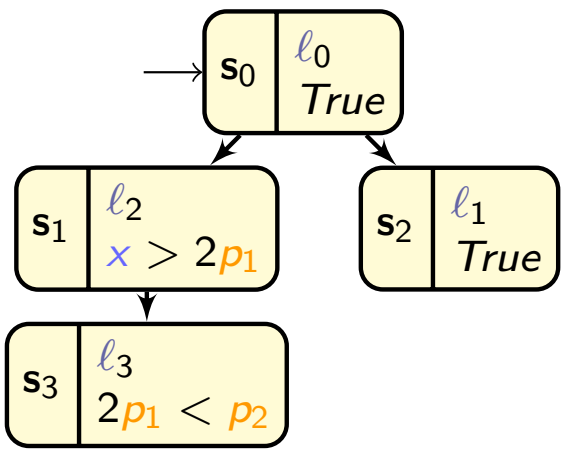

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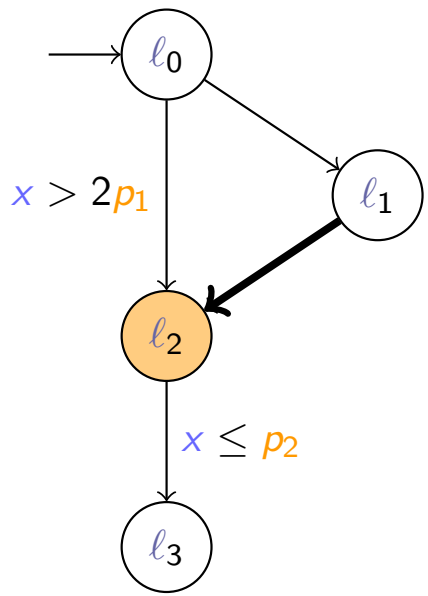

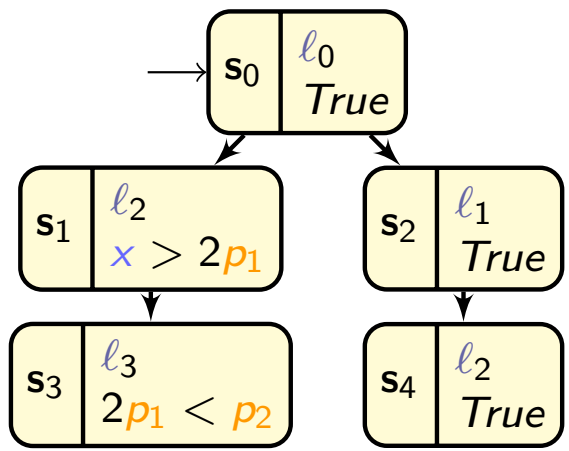

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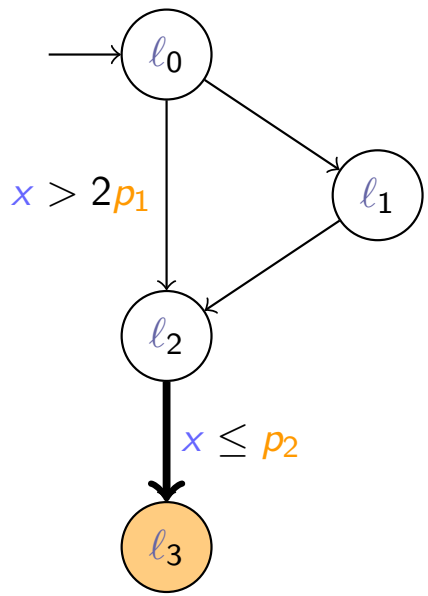

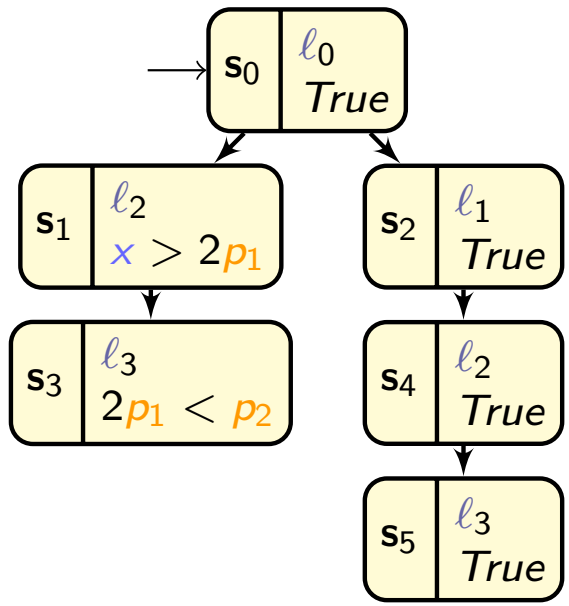

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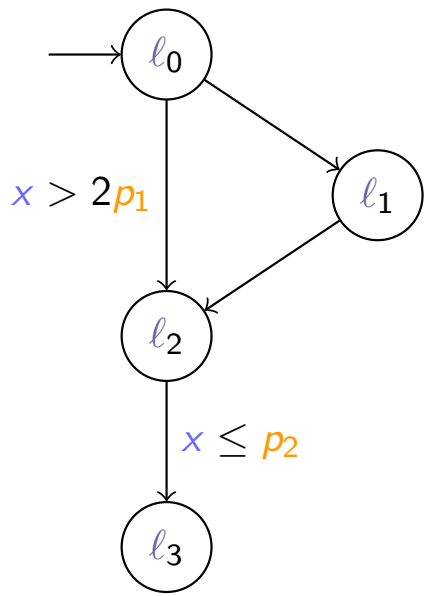

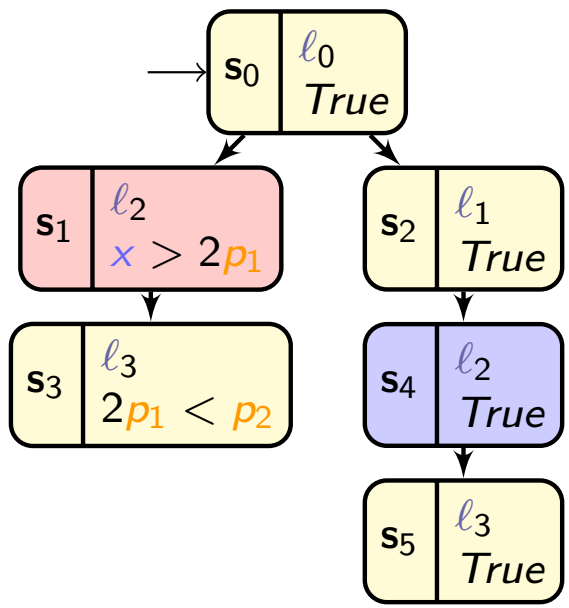

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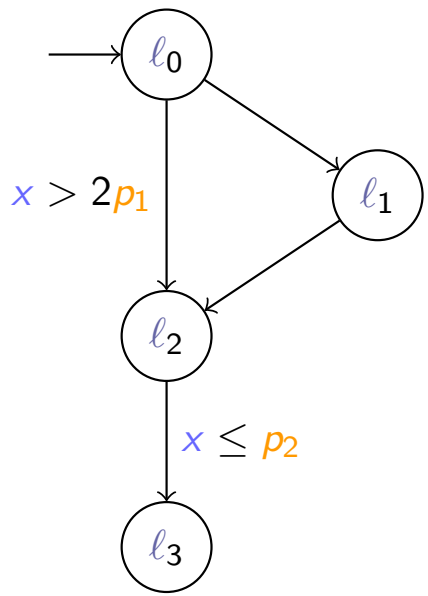

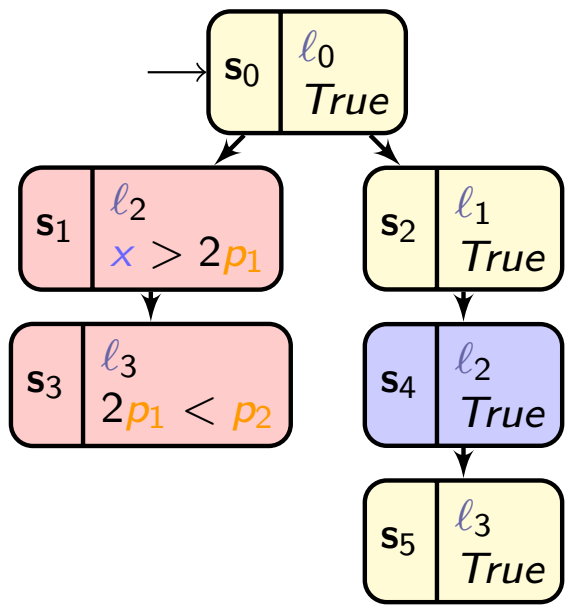

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Merging states

Definition

Two states (●, $\mathbf{C}_1$) and (●, $\mathbf{C}_2$) are mergeable if:

- ● = ●
- $\mathbf{C}_1 \cup \mathbf{C}_2$ is convex

Their merging is defined by (●, $\mathbf{C}_1 \cup \mathbf{C}_2$)

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State merging techniques were introduced:

- in TA [Dav05]
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Merging preserves reachability

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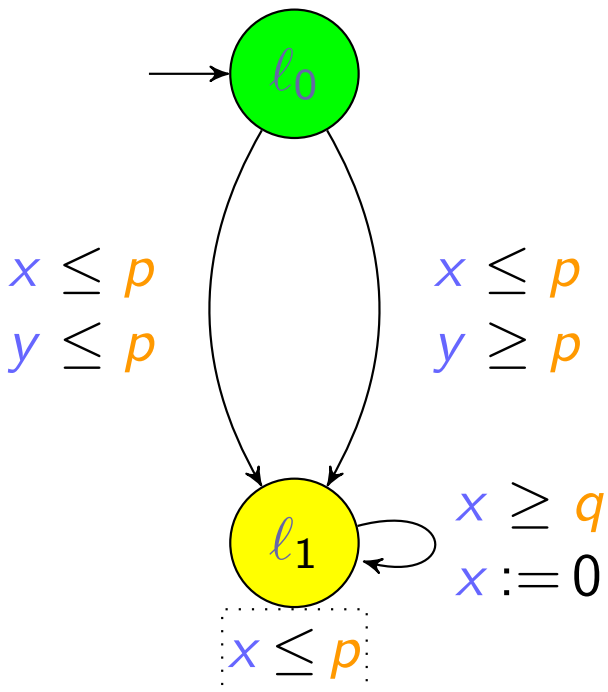
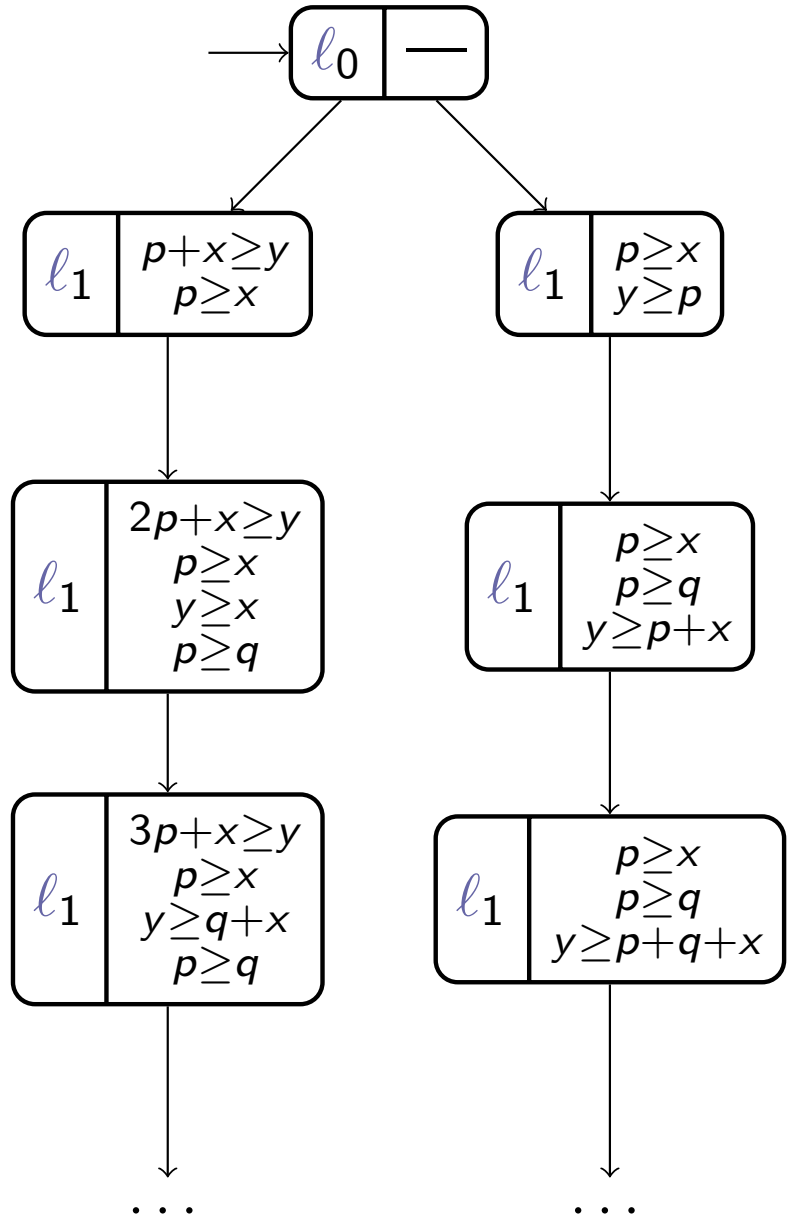
Merging can make difference for termination!

PZG without any heuristic





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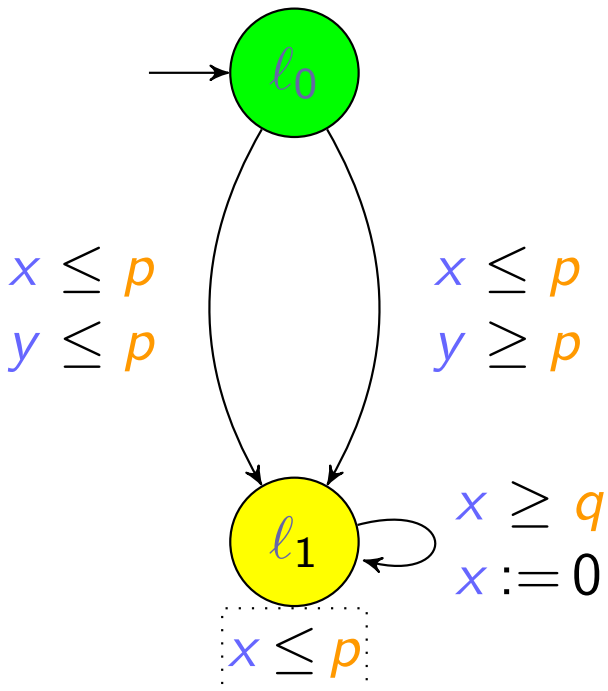
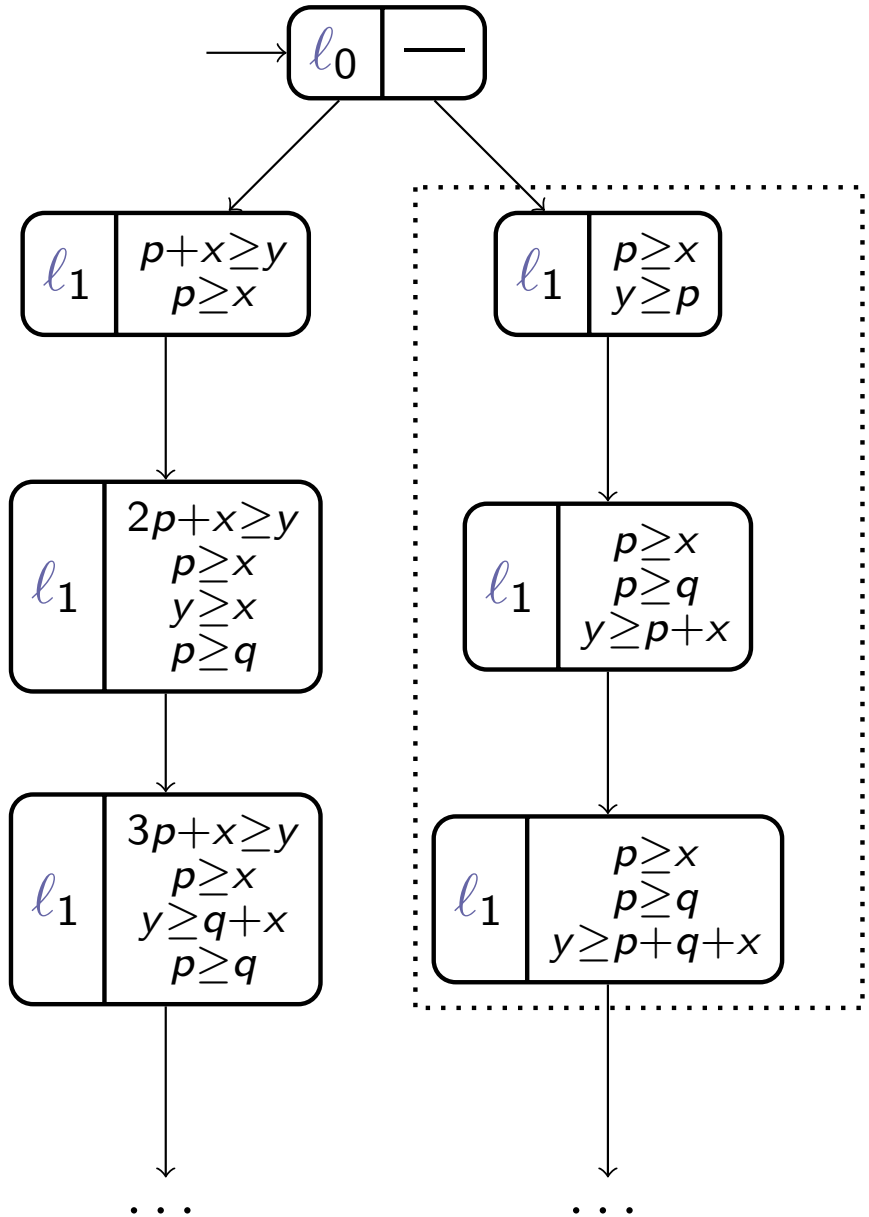
PZG with inclusion





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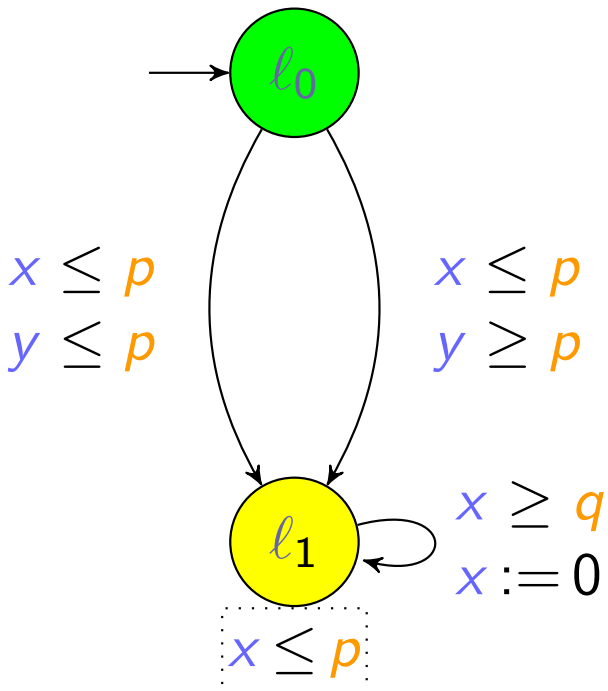

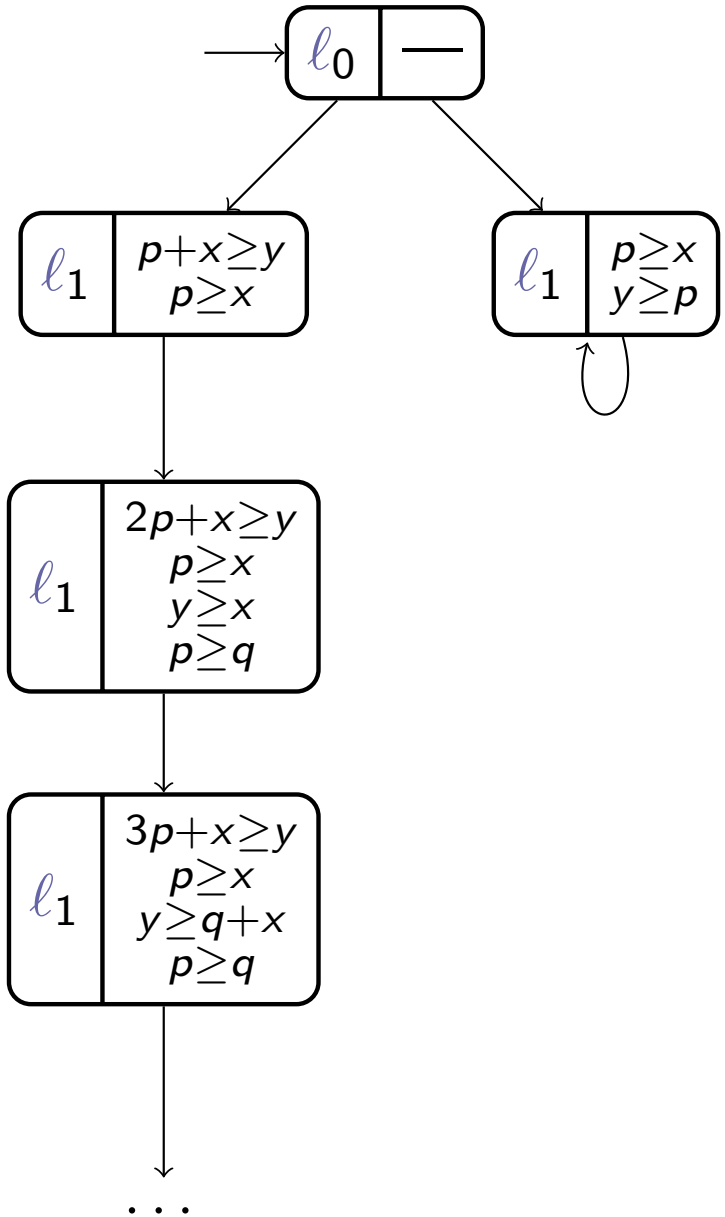

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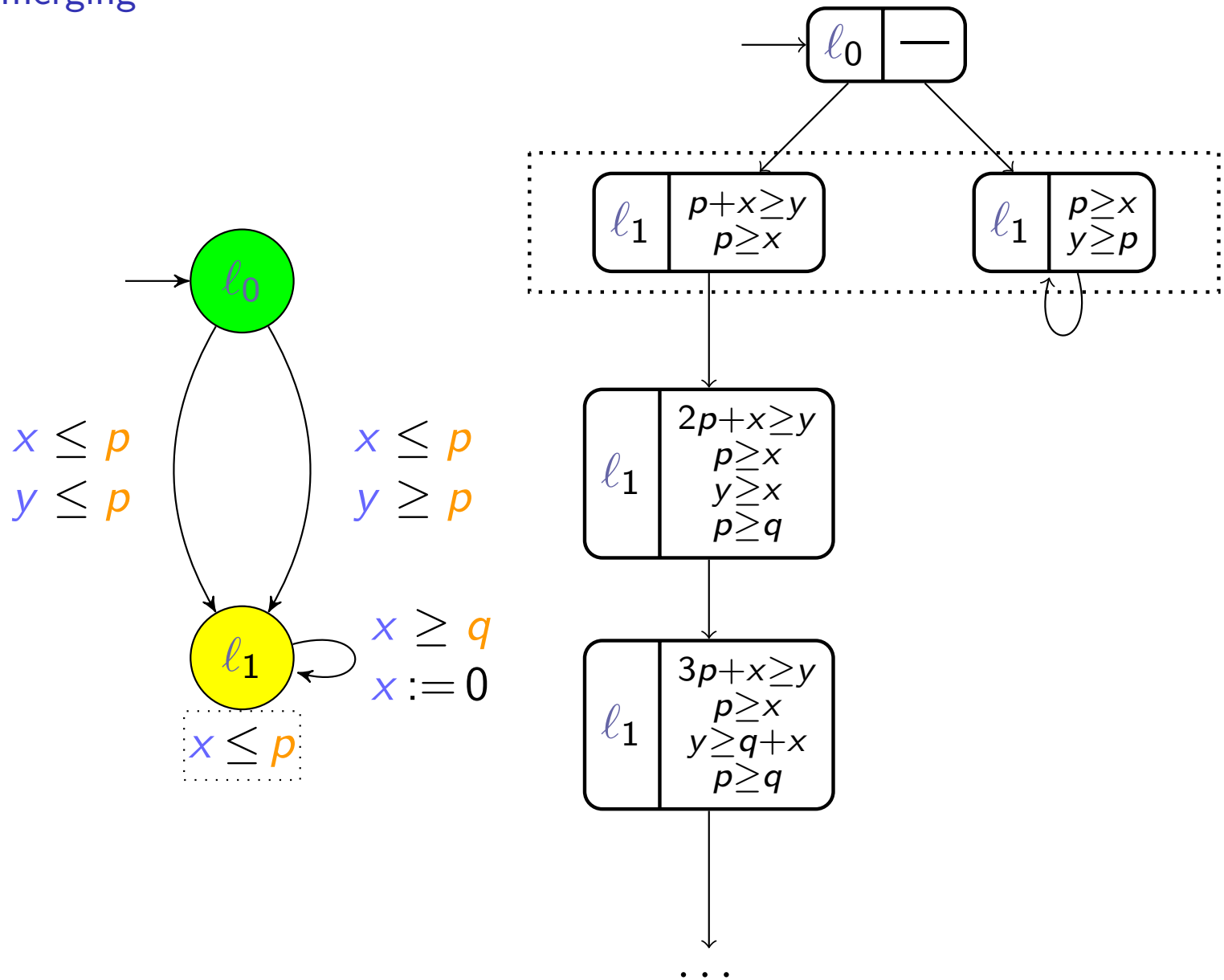
PZG with merging



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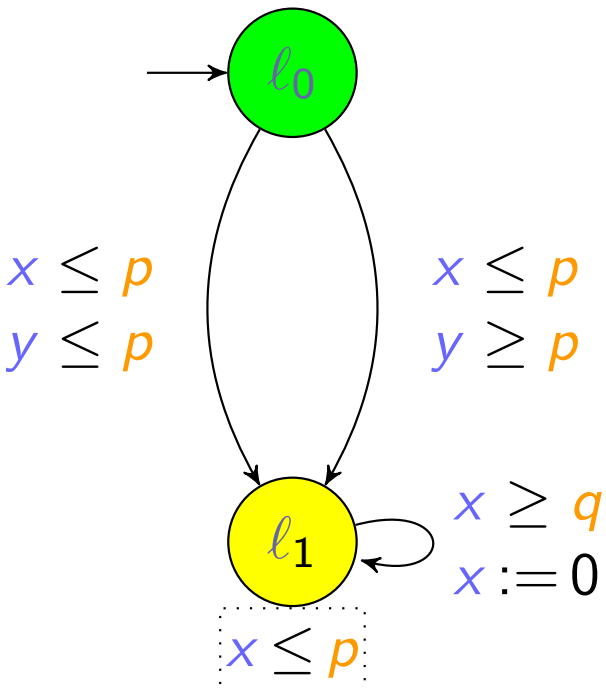
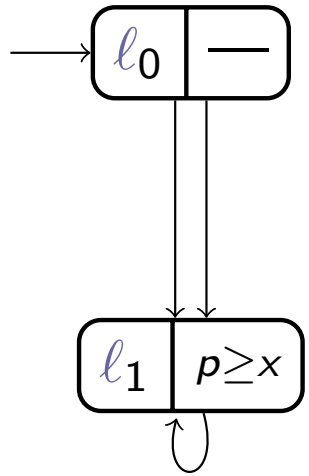




The construction of the PZG

Algorithm 1: BFS by layer layerBFS($\mathcal{A}$)

1 $\mathbf{Visited} \leftarrow \{\mathbf{s}_0\}$
2 $\mathbf{Queue} \leftarrow \{\mathbf{s}_0\}$
3 $\Rightarrow \leftarrow \emptyset$
4 **while** $\mathbf{Queue} \neq \emptyset$ **do**
5 $\mathbf{Q}_{new} \leftarrow \emptyset$
6 **foreach** $\mathbf{s} \in \mathbf{Queue}$ **do**
7 **foreach** $(e, \mathbf{s}') \in \mathrm{SuccE}(\mathbf{s})$ **do**
8 $\mathbf{Q}_{new} \leftarrow \mathbf{Q}_{new} \cup (\{\mathbf{s}'\} \setminus \mathbf{Visited})$
9 $\Rightarrow \leftarrow \Rightarrow \cup \{(\mathbf{s}, e, \mathbf{s}')\}$
10 $\mathbf{Visited}, \mathbf{Queue} \leftarrow \mathit{mergeSets}(\mathbf{PZG}, \mathbf{Visited}, \mathbf{Q}_{new})$

Heuristics for merging

What to merge with what? Queue, Visited, Ordered

Restart after merge?

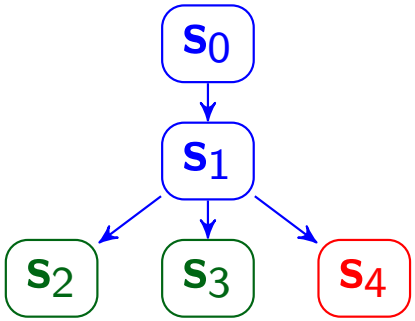
When to update the state space?

- After each merge
- After exploring the candidates list
- After exploring a BFS level

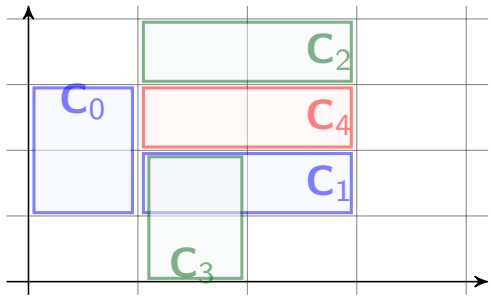
How to update the state space?

- Reconstruction of the state-space
- *In situ*

Illustration of the merging options



- visited
- in the queue
- being processed
- after merge



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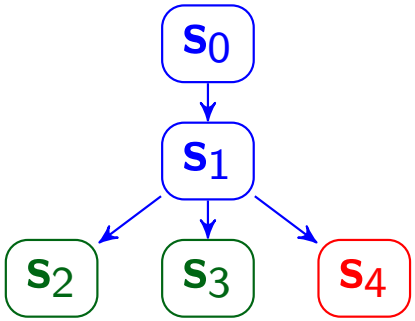

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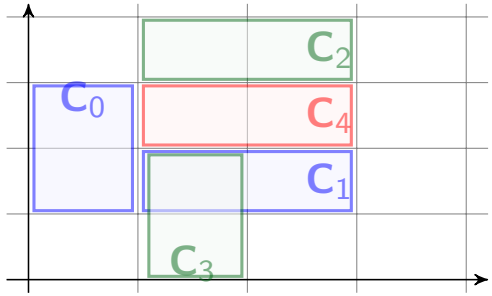


Merge with Queue

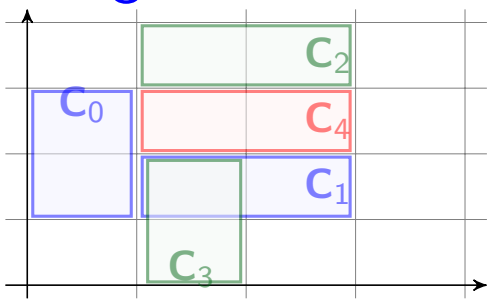
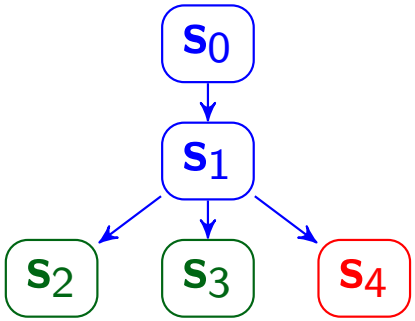


Illustration of the merging options



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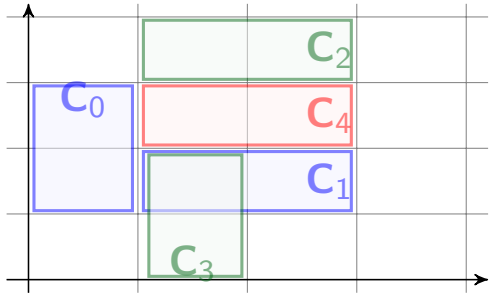

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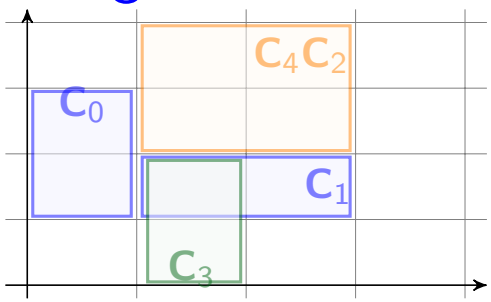

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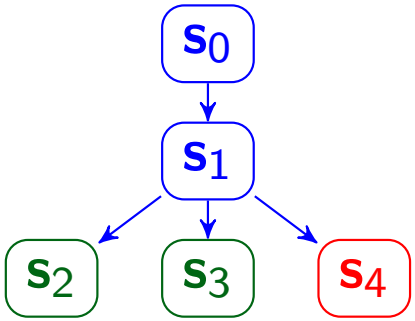

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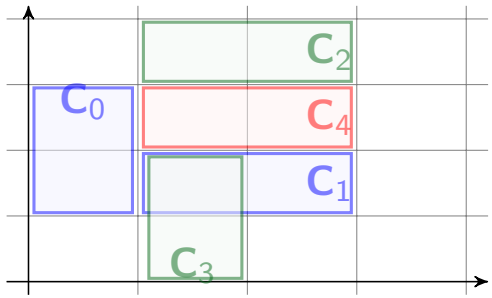

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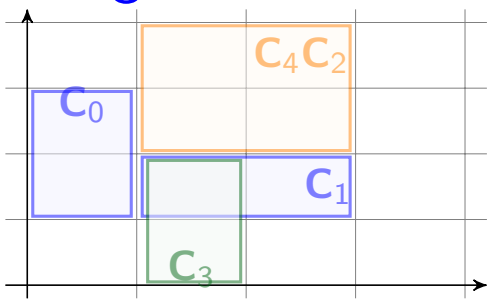

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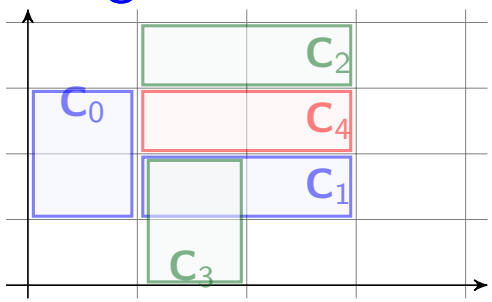

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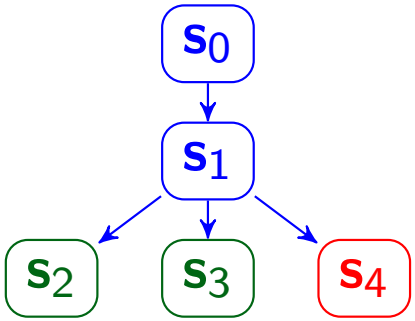

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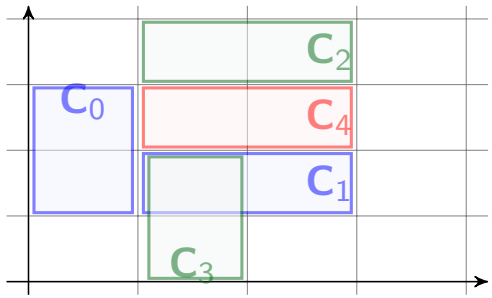

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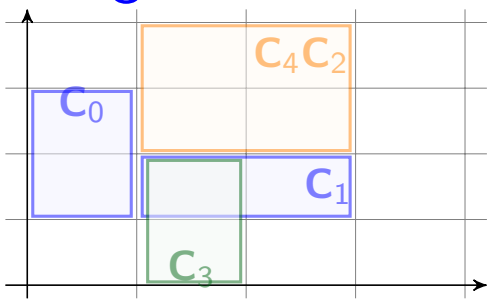

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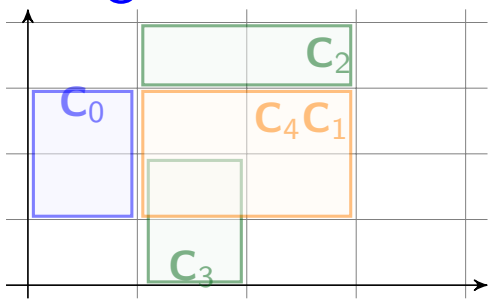

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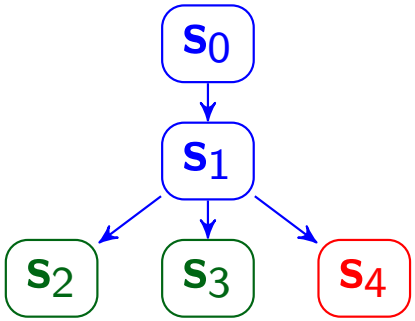

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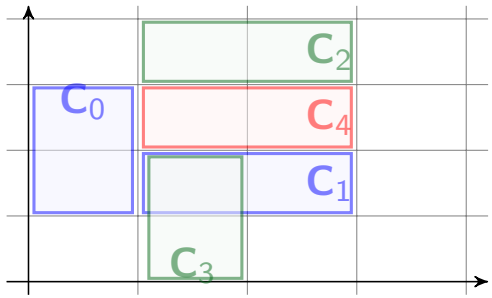

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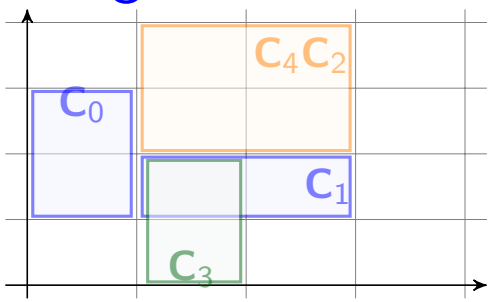

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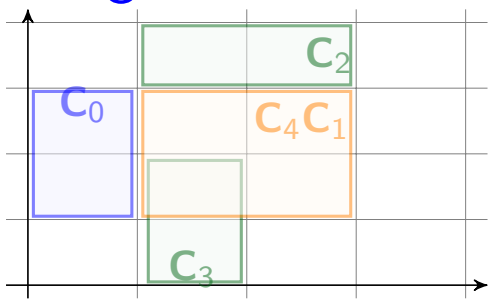

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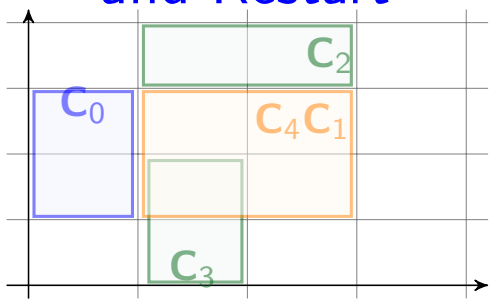
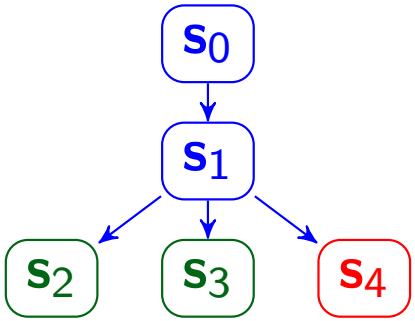


Illustration of the merging options



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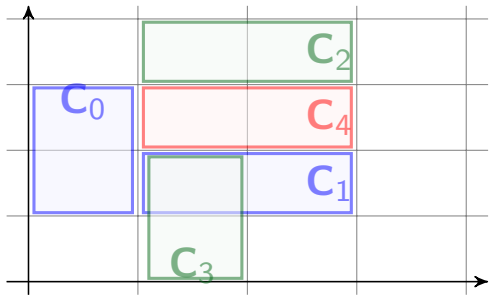

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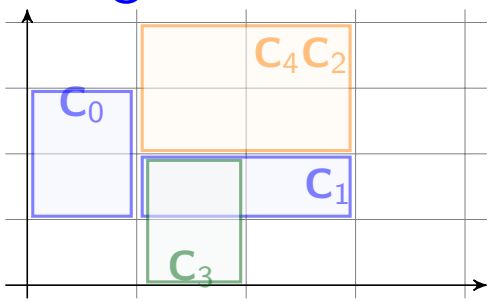

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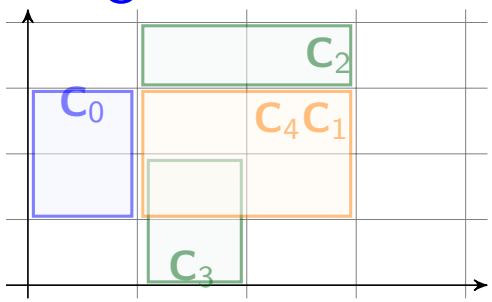

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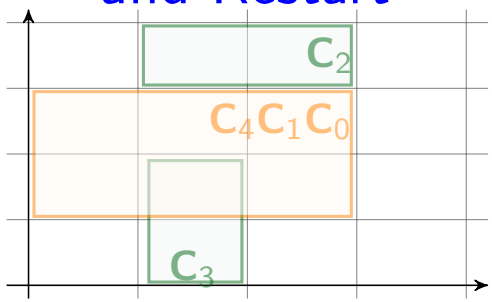

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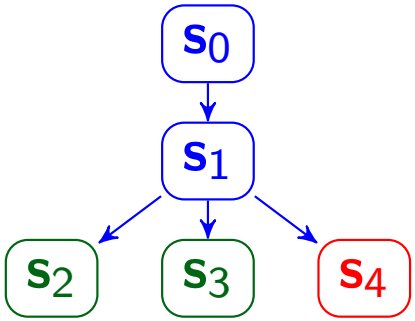

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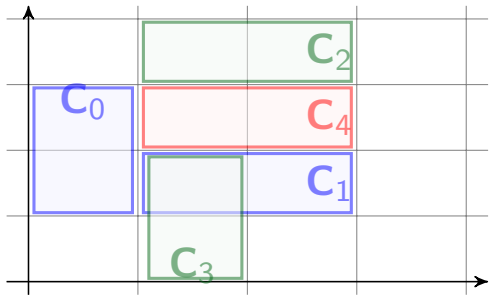

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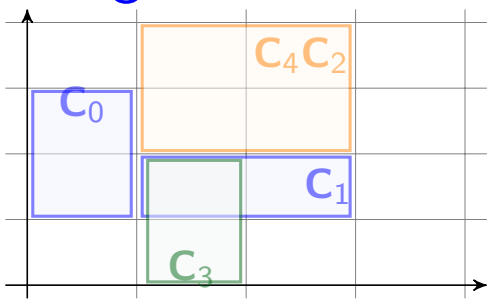

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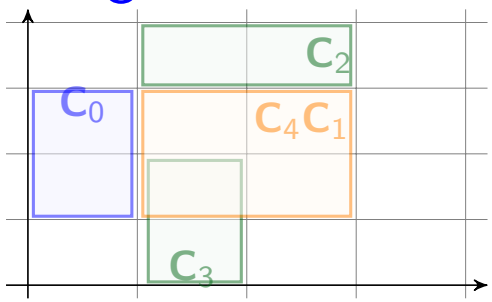

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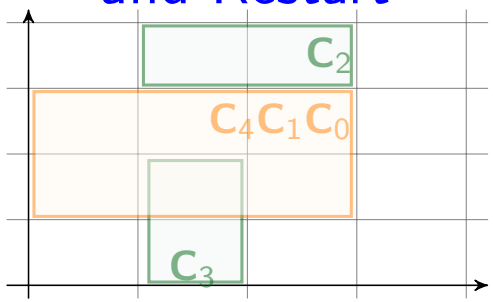

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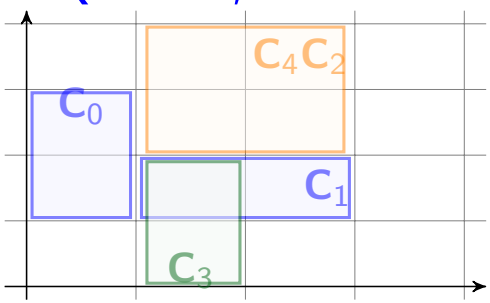

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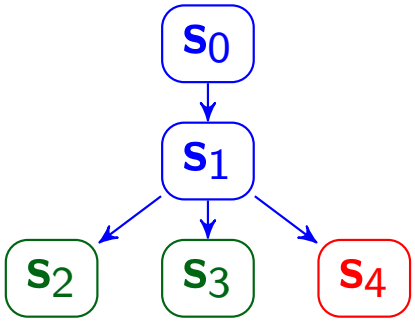

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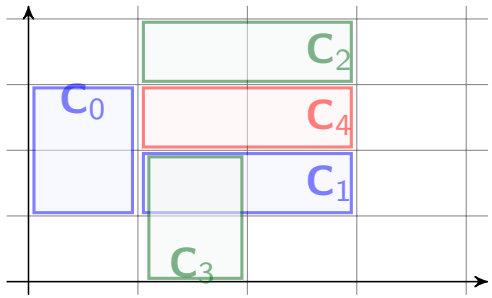

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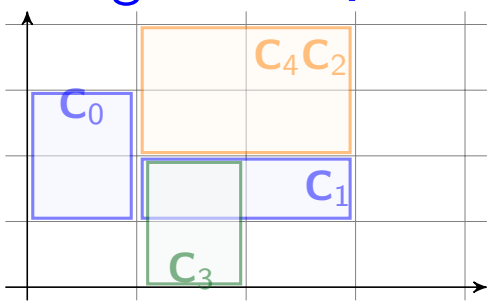

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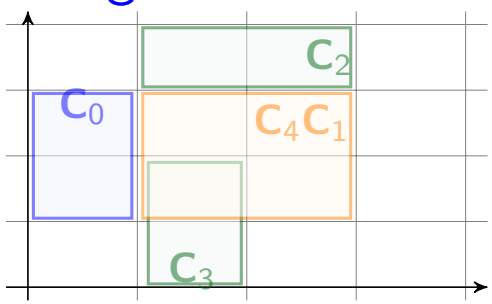

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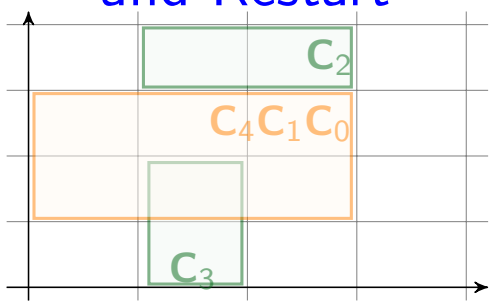

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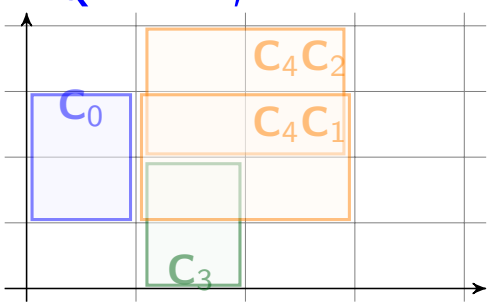

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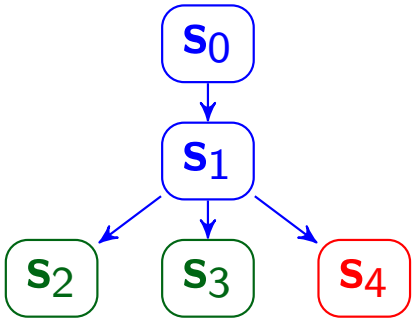

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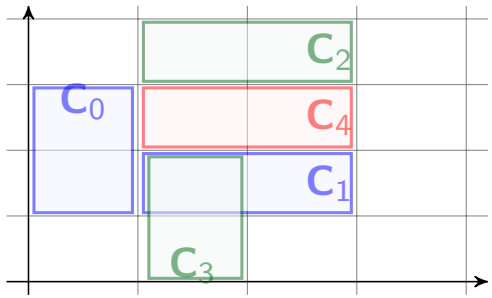

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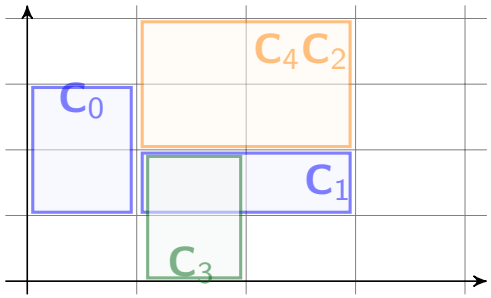

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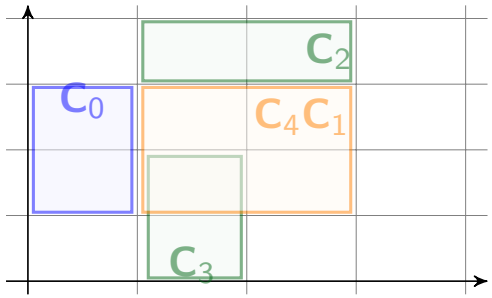

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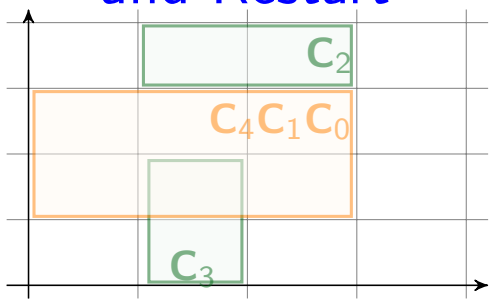

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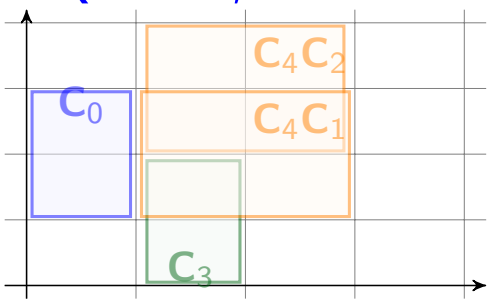
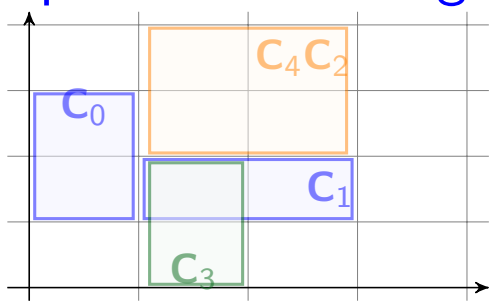


Update after Merge



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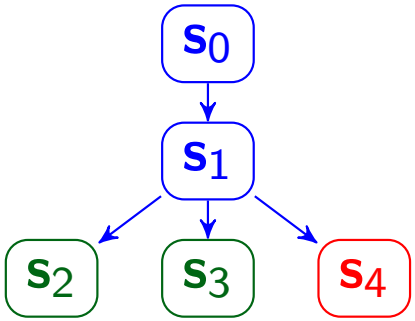

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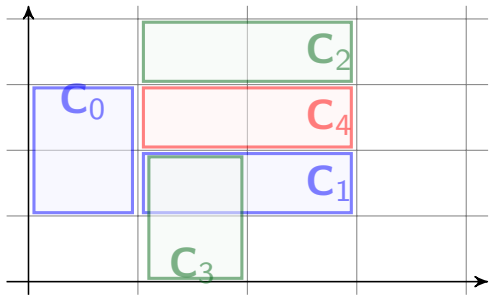

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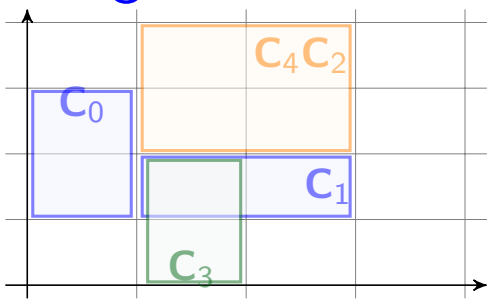

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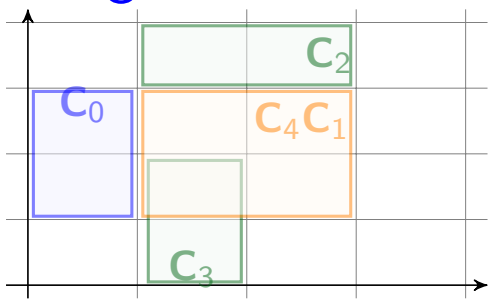

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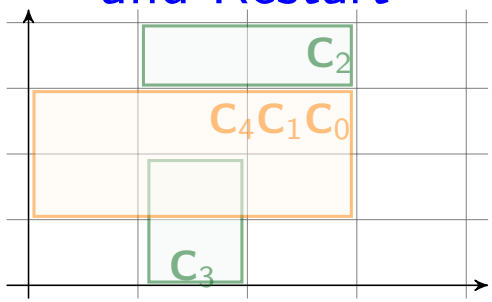

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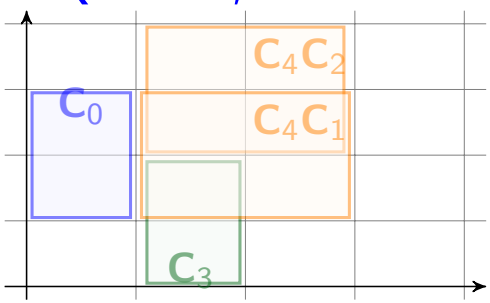
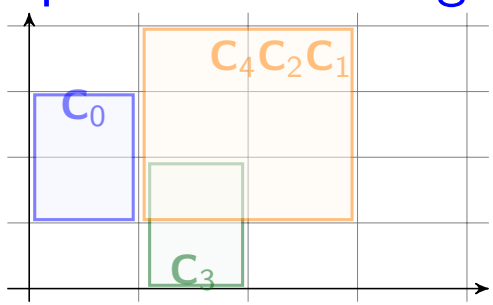


Update after Merge



Experiments

- Comparison of all the combinations of heuristics
- Use of the IMITATOR library, restricted to reachability-based properties [AMP21]:
  - 124 executions (model, reachability property)
  - 42 executions perform at least one merge

[AMP21] Étienne André, Dylan Marinho, and Jaco van de Pol. "A Benchmarks Library for Extended Timed Automata". In: *TAP* (June 21–25, 2021). Ed. by Frédéric Loulergue and Franz Wotawa. Vol. 12740. LNCS. virtual: Springer, 2021, pp. 39–50. DOI: 10.1007/978-3-030-79379-1_3

Results

		Nomerge	M2.12	RVMr	OQM
Time	# wins	24	20	22	**42**
	Avg (s)	10.0	5.47	4.56	**3.77**
	Avg (merge) (s)	18.8	7.83	5.57	**3.63**
	Avg (no merge) (s)	3.83	**3.82**	3.85	3.88
	Median (s)	1.39	1.2	1.14	**1.12**
	Norm. avg	1.0	0.91	0.91	**0.87**
	Norm. avg (merge)	1.0	0.75	0.74	**0.64**
	Norm. avg (no merge)	**1.0**	1.02	1.03	1.03
States	# wins	0	19	**37**	16
	Avg	11443.08	11096.54	**11064.37**	11120.79
	Avg (merge)	1512.02	670.43	**592.31**	729.33
	Median	2389.5	703.5	**604.5**	905.0
	Norm. avg	1.0	0.86	**0.84**	0.88

Conclusion and Perspectives

- investigated the merge operation for reachability analysis in PTAs
- proposed several heuristics
- shown they are sound
- extensive experiments to compare these approaches

Conclusion and Perspectives

- investigated the merge operation for reachability analysis in PTAs
- proposed several heuristics
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- tackle the handling of merged states:
  - pruning away
  - separate collection of potential mergers
- merge more than 2 states
- canonical merge representatives
- compatibility of merging and liveness properties

References I

[AD94] Rajeev Alur and David L. Dill. "A theory of timed automata". In: *Theoretical Computer Science* 126.2 (Apr. 1994), pp. 183–235. ISSN: 0304-3975. DOI: 10.1016/0304-3975(94)90010-8.

[AFS13] Étienne André, Laurent Fribourg, and Romain Soulat. "Merge and Conquer: State Merging in Parametric Timed Automata". In: *ATVA* (Oct. 15–18, 2013). Ed. by Dang-Van Hung and Mizuhito Ogawa. Vol. 8172. LNCS. Ha Noi, Viet Nam: Springer, Oct. 2013, pp. 381–396. DOI: 10.1007/978-3-319-02444-8_27.

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[AMP21] Étienne André, Dylan Marinho, and Jaco van de Pol. "A Benchmarks Library for Extended Timed Automata". In: *TAP* (June 21–25, 2021). Ed. by Frédéric Loulergue and Franz Wotawa. Vol. 12740. LNCS. virtual: Springer, 2021, pp. 39–50. DOI: 10.1007/978-3-030-79379-1_3.

[Dav05] Alexandre David. "Merging DBMs Efficiently". In: *NWPT* (Oct. 19–21, 2005). DIKU, University of Copenhagen, 2005, pp. 54–56.

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