Computing the connected components of real algebraic curves

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Joint work with Mohab Safey El Din

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Problem Statement

Real Algebraic Curves

$\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{C}^n, \qquad f_1, \ldots, f_r \in \mathbb{Q}[x_1, \ldots, x_n]$

$\mathcal{C}_{\mathbb{C}} := V(f_1, \ldots, f_r) = \{\mathbf{x} \in \mathbb{C}^n \mid f_1(\mathbf{x}) = \cdots = f_r(\mathbf{x}) = 0\}, \quad \dim \mathcal{C}_{\mathbb{C}} = 1.$

$\mathcal{C} = \mathcal{C}_{\mathbb{C}} \cap \mathbb{R}^n$

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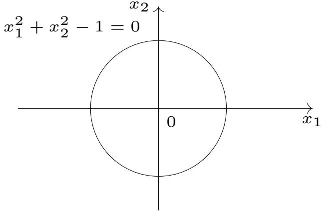
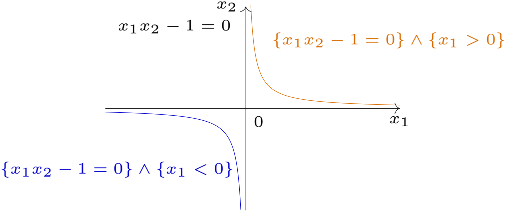
$\mathcal{C} = \mathcal{C}_{\mathbb{C}} \cap \mathbb{R}^n$

$\mathcal{C} = \bigcup_{i=1}^{m} C_i, \qquad C_i$ **connected component**

- **Finite** number of connected components: $m \leq d(2d-1)^{n-1}, \ d = \max_i \deg f_i$.
- Each C_i is **semi-algebraic** → **semi-algebraic polynomial constraints**





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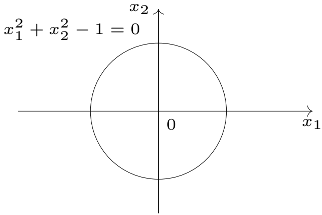
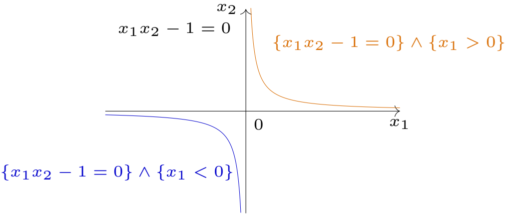




Goal: Given $\mathcal{C}$, compute the connected components C_i

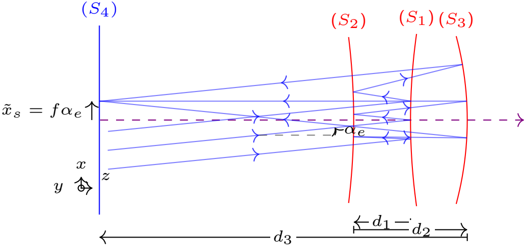
State of the Art and Motivation

Classification of **optical system** for optimal design

How lenses, mirrors and other components are combined.

[Liu-Bauer-Viard-Rolland (2021)]

[Drogoul (2025)]



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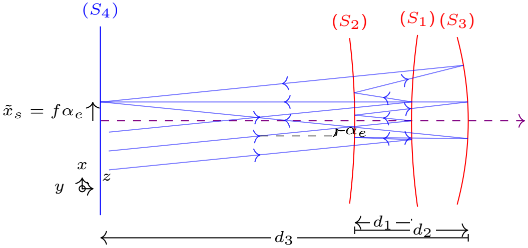

Prior work

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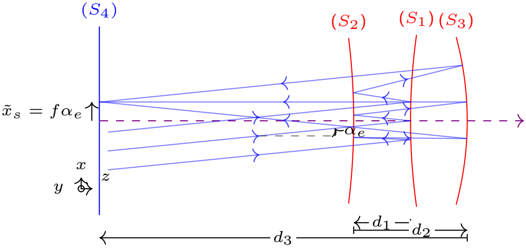

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Related work

[Cheng-Jin-Pouget-Wen-Zang (2024)] Topology of curves in $\mathbb{R}^3$ — $\widetilde{O}(d^{18} + d^{17}\tau)$

[Islam-Poteaux-Prébet (2023)] Connectivity queries space curves $(\delta \sim d^{n-1})$ — $\widetilde{O}(\delta^6 + \delta^5\tau)$

Our Contributions

Plane Curves Given $f \in \mathbb{Q}[x_1, x_2]$ square-free of magnitude (δ, τ), $\mathcal{C} = V_{\mathbb{R}}(f) \subset \mathbb{R}^2$

Main Theorem (Plane curve)

The connected components of $\mathcal{C}_{\mathbb{R}}$ can be computed in $\widetilde{O}(\delta^7 + \delta^6\tau)$ bit operations.

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$$\widetilde{O}(n^{12} d^{7n-7} (h + \overline{\tau} + 1) + (h + \log \frac{1}{\varepsilon})(n^{14} d^{6n-5} + \binom{n+d}{n} n^{16} d^{3n}))$$

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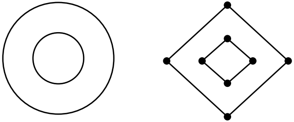
$$n = 3 \to \widetilde{O}(d^{14} + \dots) \quad \widetilde{O}(\underbrace{d^{18} + \dots}_{\text{Topology}}) \quad \widetilde{O}(\underbrace{d^{12} + \dots}_{\text{Connectivity}})$$

Plane Curves: General strategy

$f \in \mathbb{Q}[x_1, x_2]$ square-free of magnitude (δ, τ); $\mathcal{C} = V_{\mathbb{R}}(f) \subset \mathbb{R}^2$

Real Isotopy Graph →
$\widetilde{O}(\delta^6 + \delta^5\tau)$
[Mehlhorn-Sagraloff-Wang (2013)]

Encoding the topology of $\mathcal{C}$.



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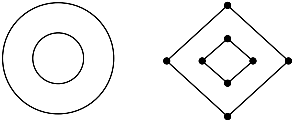

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Semi-algebraic Real Isotopy Graph
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Enrich the graph with new vertices and edges such that:

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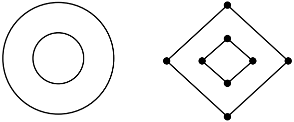

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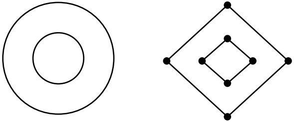

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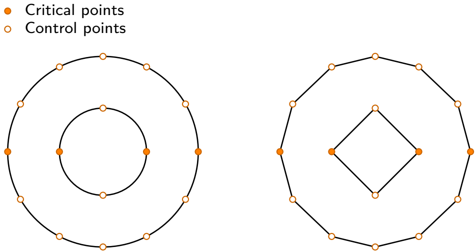
General case

1. $f = f_1 \cdots f_r$, each f_i satisfying $\mathcal{S}(f_i)$ finite: $g = \gcd(f, \partial^k_{x_2} f) \neq 1 \rightarrow \; f = g \cdot f/g$
2. Combine the informations coming from f_i's

Plane Curves: The special case

Construction when $\mathcal{S}(f)$ is finite

- Start from isotopy graph.



Plane Curves: The special case

Construction when $\mathcal{S}(f)$ is finite

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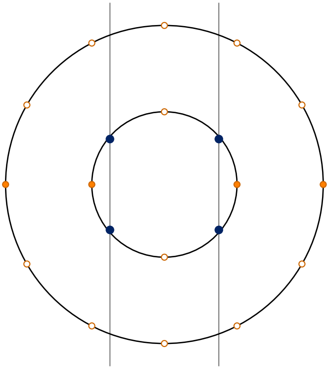
- Compute the points in $\mathcal{S}(f)$.



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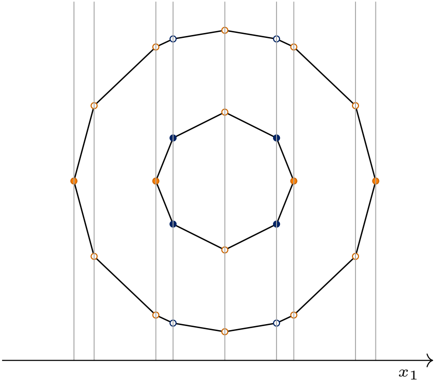

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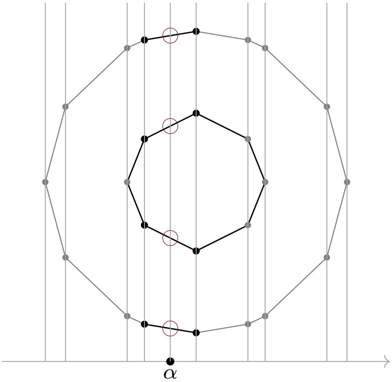
Each edge of this graph admit a semi-algebraic description:

Thom's encoding

$f(\alpha, x_2) = 0$



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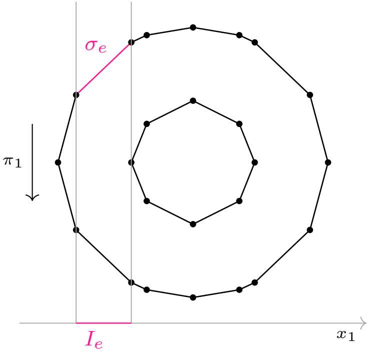

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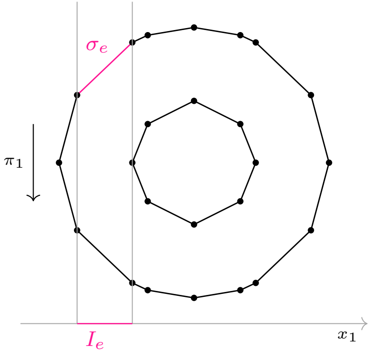

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Each connected component admits a semi-algebraic description as the union of the descriptions of its edges and vertices.

Space Curves: Main Ingredients

$\mathbf{f} = (f_1, \dots, f_{n-1}) \subset \mathbb{Q}[x_1, \dots, x_n]$ radical, $\dim = 1$; $\mathcal{C} = V_{\mathbb{R}}(\mathbf{f}) \subset \mathbb{R}^n$

Birational projection $\pi_{u,v} : \mathcal{C} \dashrightarrow \mathcal{C}_2$

One-dimensional parametrization $\mathcal{C}_2 = V\Big(w(u,v)\Big) \quad \pi_{u,v}^{-1} : \mathcal{C}_2 \dashrightarrow \mathcal{C}$

[Giusti-Lecerf-Salvy (2001)]

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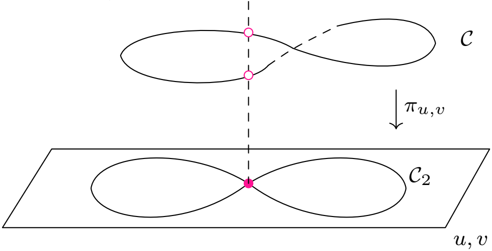

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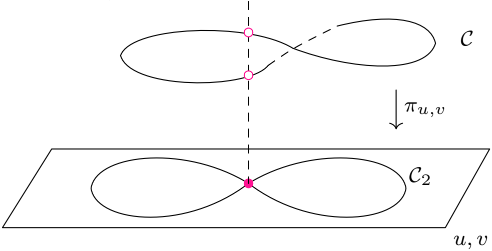


Generic projection (A_1)
[Islam-Poteaux-Prébet (2023)]

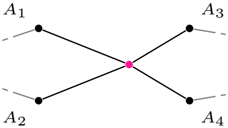
$$\boxed{\mathrm{app}(\mathcal{C}_2) \subset \mathrm{sing}(\mathcal{C}_2)}$$

- Connectivity of $\mathcal{C}$
- Nodes of $\mathcal{C}_2$
- 2 preimages

Space Curve: Strategy

Semi-algebraic isotopic graph of $\mathcal{C}_2$

- Each edge admits a semi-algebraic description.
- Connectivity of $\mathcal{C}_2$.



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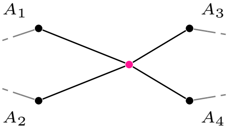
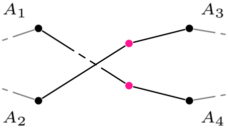


Lifted graph

- Each edge still has a semi-algebraic description of a branch of $\mathcal{C}_2$.
- Connectivity of $\mathcal{C}$.



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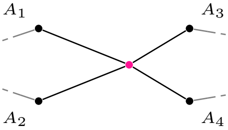

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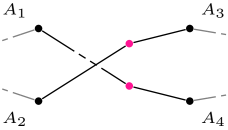

Lift descriptions of edges and vertices (outside $\pi_{u,v}^{-1}(\mathrm{app}(\mathcal{C}_2))$)
one-dimensional parametrization + $f_1, \ldots, f_{n-1}$

↓

Semi-algebraic description of the connected components of $\mathcal{C} \setminus \pi_{u,v}^{-1}(\mathrm{app}(\mathcal{C}_2))$

Missing Points: $\pi_{u,v}^{-1}(\mathrm{app}(\mathcal{C}_2))$

Space Curves: Recovering the missing points

Idea: use a second generic projection onto a different plane.

First projection (choice of $\mathbf{A}_1$): $\pi_{u,v} : \mathcal{C} \dashrightarrow \mathcal{C}_2$

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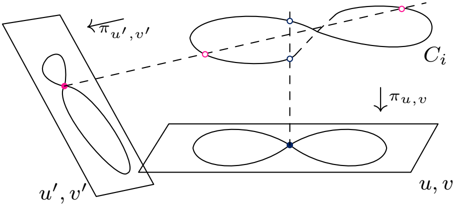
Theorem

Generic choice of $\mathbf{A}_2 \;\Rightarrow\; \pi^{-1}_{u',v'}\big(\mathrm{app}(\mathcal{C}_2')\big) \,\cap\, \pi^{-1}_{u,v}\big(\mathrm{app}(\mathcal{C}_2)\big) = \varnothing.$



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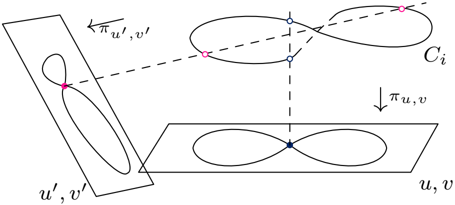


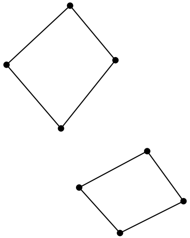
For each C_i connected component we have two descriptions

- $C_i \setminus \left(\pi_{u,v}^{-1}(\mathrm{app}(\mathcal{C}_2)) \cap C_i\right)$
- $C_i \setminus \left(\pi_{u',v'}^{-1}(\mathrm{app}(\mathcal{C}_2')) \cap C_i\right)$

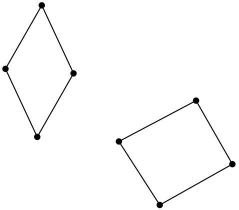
Space Curves: Linking the two descriptions

Problem: Match the descriptions correspond to the same connected component of $\mathcal{C}$

First Projection



Second Projection



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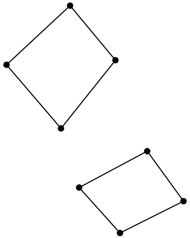
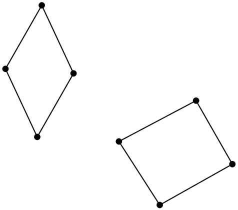


Second Projection



$\mathcal{P} \subset \mathrm{Reg}(\mathcal{C})$ **finite**
containing at least one point in
each connected component of $\mathcal{C}$
[Safey El Din-Schost (2003)]

$\longrightarrow$

Add the set $\mathcal{P}$ to the **graphs**

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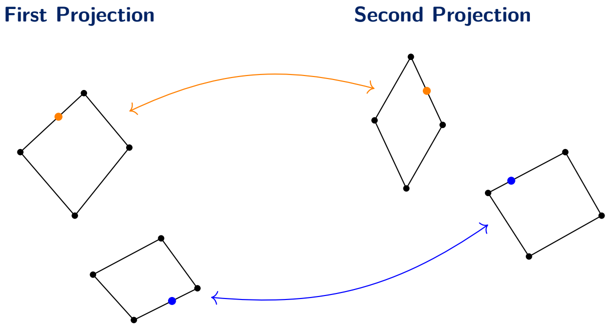

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Conclusions and Perspectives

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Thank you for your attention!