

Conjectural Variations and General Oligopoly Equilibria

Ludovic JULIEN

Introduction

- **Motivations:**

- (i) - Modelling expectations in GE under strategic interactions
- (ii) - Asymmetries in GE under SI
- (iii) - Foundations of perfect competition (Arrow (1959), Debreu (1959))

Today (iii): Can we deduce PC from an IC framework?

1. Introduction (suite)

1. *Atomic* approach (Aumann (1964)),
2. **Asymptotic approach** (Codognato-Gabszewicz (1991), (1993) and Gabszewicz-Vial (1972)),
3. *Game-Theoretic approach* (Postlewaite-Roberts (1976), Gale (2000)).

- Casting **conjectural variations** into GE
 - Idea of **CV**: to take into account the perceptions of each individual about his rivals' responses to a change in his own individual decision (Bowley (1924), Figuières et al. (2004), Julien (2006)).
 - + Locally constant conjectures (Perry (1982), Julien (2008)),
 - Idea today: **intregation of CV in the asymptotic approach.**

2. The model

- **Pure exchange economy:**

- L divisible goods $\ell = 1, \dots, L$

- H agents, indexed h , $h = 1, \dots, H$

- *Preferences:* $U(x_h) = \prod_{\ell=1}^{\ell=L} x_{h\ell}^{\alpha_\ell} \quad \alpha \in (0,1)$

- *Endowments:* $\omega_h = (0, \dots, 1, \dots, 0) \quad h = n_{\ell-1} + 1, \dots, n_\ell$

The economy (suite)

- Strategy sets:

$$S_{h\ell} = \{s_{h\ell} : 0 \leq s_{h\ell} \leq 1\}$$

- Beliefs: $\frac{\partial \sum_{-h \neq h} s_{-h\ell}}{\partial s_{h\ell}} = v_\ell \quad h = n_{\ell-1} + 1, \dots, n_\ell$

DEFINITION 1. An economy is a collection $\xi = \{X_h, (\succ_h), \omega_{h\ell}, v_\ell\}_{h=1, \dots, H}^{\ell=1, \dots, L}$.

The economy (suite)

- DEFINITION 2. A CVGE for $\xi = \{X_h, (\succ)_h, \omega_{h\ell}, S_{h\ell}, \nu_\ell\}$ is given by a vector of strategies $(\tilde{s}_{11}, \dots, \tilde{s}_{h\ell}, \dots, \tilde{s}_{HL})$ an allocation $\tilde{x} \in \mathbb{R}_+^{HL}$ and a vector of conjectural variations $\nu = (\nu_1, \dots, \nu_\ell, \dots, \nu_L)$ such that:

$$(i) \quad \tilde{x}_h = x_h((\tilde{s}_{h\ell}(\nu_\ell), \tilde{s}_{-h\ell}(\nu_\ell)), \forall h$$

$$(ii) \quad \sum_h \tilde{x}_{h\ell}(\tilde{p}(\tilde{s}_\ell), \tilde{s}_{h\ell}(\tilde{s}_\ell)) = \sum_h \omega_{h\ell}, \forall \ell$$

$$(iii) \quad U_h(\tilde{x}_h(\tilde{p}(\tilde{s}_{h\ell}(\nu_\ell)), \tilde{s}_{h\ell}(\nu_\ell), \tilde{s}_{-h\ell}(\nu_\ell))) \geq \\ U_h(x_h(p, s_{h\ell}(\nu_\ell), \tilde{s}_{-h\ell}(\nu_\ell))), \forall h$$

3. Market prices and strategic plans

- Market clearing prices:
$$\left(\frac{p_\ell}{p_k} \right) = \frac{\alpha_\ell (1 - \alpha_\ell) \sum_h s_{hk}}{\alpha_k (1 - \alpha_k) \sum_h s_{h\ell}}$$
- Strategic plans:

$$\text{Arg max} \prod_{\substack{k=1 \\ k \neq \ell}}^{k=L} \left(\frac{\alpha_\ell}{1 - \alpha_k} s_k \right)^{\alpha_k} \left\{ \frac{(1 - s_{h\ell})^{\alpha_\ell} (s_{h\ell})^{1 - \alpha_\ell}}{[s_{h\ell} + (n_\ell - n_{\ell-1} - 1)s_{-h\ell}]^{1 - \alpha_\ell}} \right\}$$

4. Equilibrium strategies & allocations

$$\tilde{s}_{h\ell} = \frac{(1 - \alpha_\ell)[n_\ell - n_{\ell-1} - (1 + \nu_\ell)]}{n_\ell - n_{\ell-1} - (1 - \alpha_\ell)(1 + \nu_\ell)}$$

$$\tilde{x}_{h\ell} = \frac{\alpha_\ell(n_\ell - n_{\ell-1})}{n_\ell - n_{\ell-1} - (1 - \alpha_\ell)(1 + \nu_\ell)}$$

$$\tilde{x}_{hk} = \frac{\alpha_\ell(1 - \alpha_\ell)(n_k - n_{k-1})[n_k - n_{k-1} - (1 + \nu_k)]}{\alpha_k(n_\ell - n_{\ell-1})[n_k - n_{k-1}][n_k - n_{k-1} - (1 - \alpha_k)(1 + \nu_k)]}$$

5. Main results

PROPOSITION 1. *When $v_\ell = -1, \forall \ell$ the conjectural general equilibrium coincides with the competitive equilibrium.*

No replication procedure or asymptotic identification.

Main results (suite)

DEFINITION 3. A locally consistent CGE for ξ is a CGE for the vector of conjectures $\nu = (\nu_1, \dots, \nu_\ell, \dots, \nu_L)$ such that if $(\tilde{s}_{11}, \dots, \tilde{s}_{h\ell}, \dots, \tilde{s}_{HL})$ is solution to $s_{h\ell} \in \text{Arg max } V_h$ then:

$$\frac{\partial \sum_{-h \neq h} s_{-h\ell}(\tilde{s}_{h\ell}, \nu_\ell)}{\partial s_{h\ell}} = \nu_\ell, \forall \ell.$$

PROPOSITION 2. The Competitive equilibrium is a locally consistent conjectural general equilibrium.

Perspectives

Asymmetries in GE under strategic interactions

Generalization with functional forms

Learning

Economic Policy