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A generalized oligopoly model with conjectural variations

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1. Introduction

- **Motivations**:

(i) – To caracterize the equilibrium of a general static oligopoly model with conjectural variations, in which the standard outcomes (Cournot, Stackelberg, perfect competition, collusion etc…) are some special cases,

(ii) – To study the effects and the role played by conjectural variations.

Introduction (1)

- **The framework:**

  - Extend the two-step oligopoly equilibrium with many leaders and followers (Daughety (1990)),

  - Arbitrary number of firms, divided in T cohorts (Boyer-Moreaux (1985), Watt (2002)), and playing sequentially among cohorts, and simultaneously within cohorts. In each cohort, each firm forms conjectural variations.

  - Market clears when all choices have been done. Generalization of standard oligopoly models.

  - Limitative features: Marginal costs are constant, market demand is linear and exogenous (standard assumptions in oligopoly models). The position of each firm in the decision sequence is exogenous.

- ***Why introducing Conjectural variations ?***

  - To capture several degrees of market power in a unified framework (Dixit 1986),

  - To study their effects on equilibrium market outcome and welfare (Figuières et al. (2004a), (2004b)).

Introduction (2)

Results:

1. Market values are determined in case of several degrees of competition,

2. Effects of conjectural variations on welfare are specified.

3. Some results about the generalized Stackelberg equilibrium

2. The model

- Consider an economy endowed with T cohorts who play sequentially, n_i^t being the number of firms in the cohort i (i=1,…,T).

- The total number of firms in the economy is $\sum_{i=1}^{T} n_i^t$

$$n_1^1, n_1^2, \ldots, n_1^{t-1}, n_1^t$$

$$n_2^1, n_2^2, \ldots, n_2^{t-1}, n_2^t$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$n_T^1, n_T^2, \ldots, n_T^{t-1}, n_T^t$$

$$n_1^1, n_1^2, \ldots, n_1^{t-1}, n_1^t$$ **Cournot**

$$n_2^1, n_2^2, \ldots, n_2^{t-1}, n_2^t$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$n_T^1, n_T^2, \ldots, n_T^{t-1}, n_T^t$$

Cournot

Stackelberg

$$n_1^1, n_1^2, \ldots, n_1^{t-1}, n_1^t$$

$$n_2^1, n_2^2, \ldots, n_2^{t-1}, n_2^t$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$n_T^1, n_T^2, \ldots, n_T^{t-1}, n_T^t$$

The model (1)

- Inverse market demand function:

$$p = \max\left\{0, a - b\sum_{t=1}^{T} X_t\right\},$$

$$X_t = \sum_{i=1}^{n_t} x_t^i.$$

2. The model (2)

- Costs:

$$c_t^i = c, \forall i, \forall t$$

Marginal costs are constant.

2. The model (3)

- Conjectural variations:

$$\frac{\partial \sum_{-i} x_t^{-i}}{\partial x_t^i} = v_t^i, \forall i$$

$$v_t^i = v_t, \forall i$$

- Coincidence with the Cournot-Nash behavior when

$$v_t = 0$$

3. Oligopoly equilibrium

DEFINITION. An oligopoly equilibrium is a sequence of equilibrium strategies $\{\tilde{x}_t\}_{t=1}^{T}$, a market price $\tilde{p}$ and a vector of conjectural variations $v = (v_1, \ldots, v_T)$ such that $\tilde{x}_t^i \in \arg\max \pi_t^i(x_t^i), \forall i, \forall t$.

$$Max\,\pi_t^i = \left\{a - b\left[x_t^i + X_t^{-i} + \sum_{\tau=1}^{t-1} X_{t-\tau} + \sum_{\tau=1}^{T-t} X_{t+\tau}(x_t^i, x_t^{-i})\right]\right\} x_t^i - c x_t^i$$

EconomiX

3. Oligopoly equilibrium (1)

$$Max\,\pi_t^i = \left\{a - b\left[x_t^i + X_t^{-i} + \sum_{\tau=1}^{t-1} X_{t-\tau} + \sum_{\tau=1}^{T-t} X_{t+\tau}(x_t^i, x_t^{-i})\right]\right\}x_t^i - c x_t^i$$

Output of cohort t

3. Oligopoly equilibrium (2)

$$Max\,\pi_t^i = \left\{a - b\left[x_t^i + X_t^{-i} + \sum_{\tau=1}^{t-1} X_{t-\tau} + \sum_{\tau=1}^{T-t} X_{t+\tau}(x_t^i, x_t^{-i})\right]\right\} x_t^i - c x_t^i$$

Output of cohort t

Output of previous cohorts (leaders)
Taken as given for cohort t

3. Oligopoly equilibrium (3)

$$Max\,\pi_t^i = \left\{a - b\left[x_t^i + X_t^{-i} + \sum_{\tau=1}^{t-1} X_{t-\tau} + \sum_{\tau=1}^{T-t} X_{t+\tau}(x_t^i, x_t^{-i})\right]\right\} x_t^i - c x_t^i$$

Output of cohort t

Output of remaining cohorts
Depends on the cohort t

Output of previous cohorts (leaders)
Taken as given for cohort t

4. Results

<u>**PROPOSITION 1**</u>. The strategic supply is:

$$\tilde{x}_t^i = \left(\frac{a-c}{b}\right)\prod_{\tau=1}^{t}\left(\frac{1+v_{\tau-1}}{n_\tau+1+v_\tau}\right), \forall t$$

It decreases with the rank of the cohort, and the number of firms in the cohort.

4. Results

<u>**COROLLARY 1**</u>. The equilibrium price is:

$$\tilde{p} = a - (a - c)\sum_{t=1}^{T} n_t \prod_{\tau=1}^{t} \left(\frac{1 + v_{\tau-1}}{n_\tau + 1 + v_\tau} \right).$$

4. Results

<u>**COROLLARY 2**</u>. The equilibrium profit is:

$$\tilde{\pi}_t = \frac{(a-c)^2}{b(1+n_t+v_t)} \Pi_{\tau=1}^{T} \left(\frac{1+v_\tau}{1+n_\tau+v_\tau} \right) \Pi_{\tau=1}^{t} \left(\frac{1+v_{\tau-1}}{1+n_{\tau-1}+v_{\tau-1}} \right).$$

Results (Suite)

PROPOSITION 2

- When $v_t = -1$ the oligopoly equilibrium coincides with the competitive equilibrium.
- When $v_t = 0, \forall t$ the oligopoly equilibrium coincides with the Stackelberg equilibrium (Cournot within each cohort).
- When $v_t = n_t - 1, \forall t$ the oligopoly equilibrium coincides with the collusive equilibrium (within each cohort).

Results (Suite)

PROPOSITION 3

The competitive equilibrium is a locally consistent oligopoly equilibrium.

The aggregate equilibrium condition coincides with any individual response.

Results (Suite)

PROPOSITION 4

Consider $\tilde{X}_t = \sum_{i=1}^{n_t} \tilde{x}_i^t$. Then $\dfrac{\partial \tilde{X}_t}{\partial v_t} < 0$

and $\dfrac{\partial \tilde{X}_{t+\tau}}{\partial v_t} > 0$ for $\tau \neq 0, \forall t$.

Results (Suite)

PROPOSITION 5

The Stackelberg (competitive and collusive), Cournot (competitive and collusive) and perfect competition equilibria can be welfare ranked (using agregate profits as a criterion).

$$W^{CCL} > W^{SCL} > W^{CCP} > W^{SCP} > W^{PCP}$$

1. The ranking is the same using profits per cohort as the criterion of welfare.

2. For a given form of competition (collusive, competitive), all firms, whatever their cohorts, prefer to play simultaneously (except for the first cohort if it is composed of very few firms).

3. Given the linear specification of the demand function, the welfare ranking of consumer surplus is exactly the opposite.

Conclusion

- Given some assumptions on the economy and the technology, we have provided a general framework for analyzing interactions in oligopolistic situations with believes.

- The timing positions should be endogenized.