Invariance in Growth Theory and Sustainable Development

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Robert Solow (1993)

`` If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that thing is…"

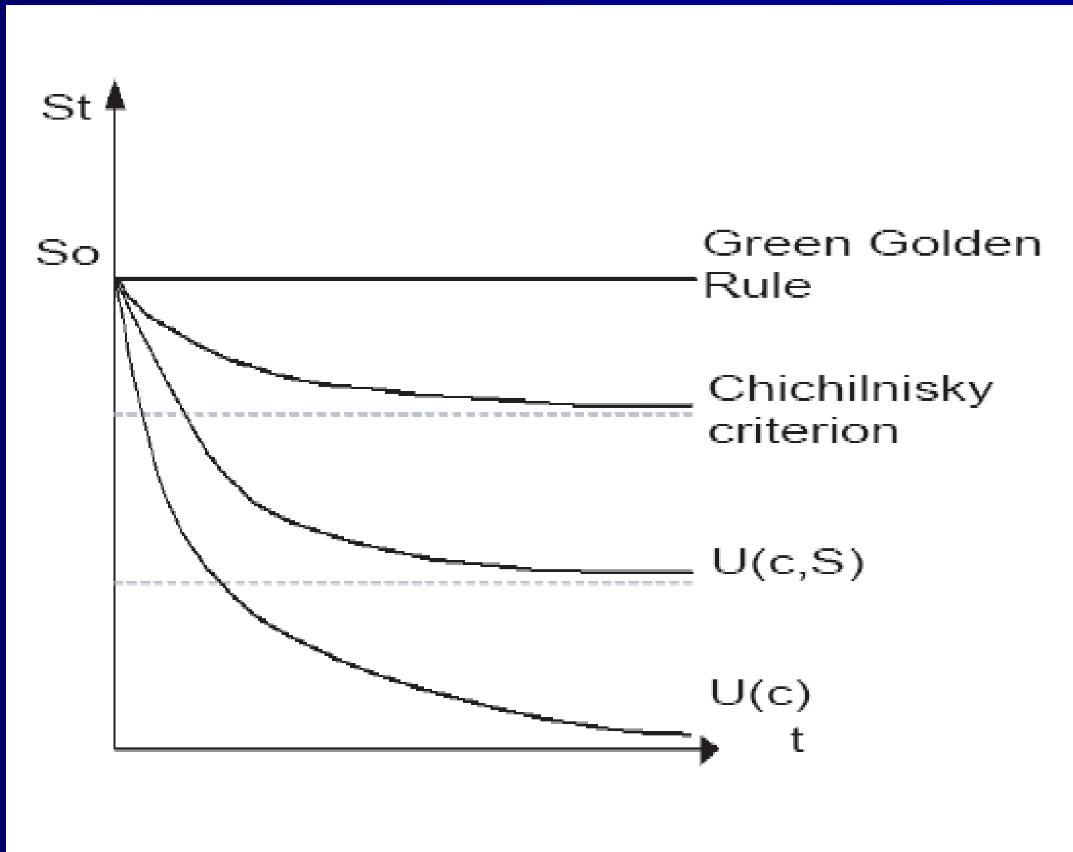
What could be conserved for sustainability ?

- Strong Sustainability: Critical natural resources
  - But how to list them?
  - At which cost should they be conserved?
- Weak Sustainability: debate on sustainability criteria
  - Abstract way to define what to be conserved

Sustainability Criteria

- Discounted utilitarian criterion $\max_{c(.)} \int_0^\infty \Delta(t) U(c_t, S_t) dt$
- Maximin criterion $\max_{c(.)} \left(\min_t U_t \right)$
- Green Golden Rule $\max_{c(.)} \lim_{t \to \infty} U(c_t, S_t)$
- Chichilnisky's criterion $W = \alpha \int_0^\infty u(c_t, s_t)\ \Delta(t)\ dt + (1 - \alpha) \lim_{t \to \infty} u(c_t, s_t)$

Resource preservation under various criteria



Sustaining an utility level

- The most used criterion: discounted utilitarianism
  - Dictatorship of the present
- Added constraint: to sustain the utility level (Asheim et al. 2001, *JEEM*)
- The criterion then has solutions under restrictive conditions

Sustainability concern

- The method is criticized (Krautkraemer 1998, *JEL* ; Cairns and Long 2006, *EDE*)
- The criterion has no solution in various cases
- If there is no solution, may be the sustainability requirement is too strong

  - ➢ What can be conserved for sustainable development ?

Purpose of the paper

- Define what is conserved along an optimal path in an economic model with a non renewable resource
- Examine conditions of such economic conservation laws
- Interpret the conservation law in term of sustainability

The Noether (1918) theorem

- Exhibit conservation laws along optimal paths of a problem of the form

$$\max_{U(.)} \int_a^b L(X, \dot{X}, t)\,dt$$

$$\dot{X} = f(X, U, t)$$

(Symmetry properties of the problem – time and state variables transformations)

$$\bar{t} = t + \tau(t,k,s)\ \varepsilon$$
$$\bar{k} = k + \xi(t,k,s)\ \varepsilon$$
$$\bar{s} = s + \mu(t,k,s)\ \varepsilon$$

- The conserved quantity is

$$\Omega \equiv \left(L - \sum_{i=1}^{n} \dot{x}^i \frac{\partial L}{\partial \dot{x}^i} \right) \tau + \sum_{i=1}^{n} \frac{\partial L}{\partial \dot{x}^i} \xi^i .$$

A cake-eating economy

- The problem

$$\max_{c(.)} \int_0^{\infty} Z(t)U(c_t, S_t)dt$$

$$\dot{S} = -G(t)r_t$$

$D(t) = \frac{1}{G(t)}$ is the technological progress

- The conservation law

$$\Omega \equiv U - cU'_c$$

- The existence conditions

$$D(t) = \frac{1}{Z(t)}.$$

  - A constant positive discount rate requires an exponential exogenous technical progress (at the same rate)

A production-consumption model

The problem

$$\max_{c(.),r(.)} \int_0^\infty Z(t)U(c_t, S_t)dt$$

The dynamics

$$\begin{aligned} \dot{K} &= D(t)F(K_t, r_t) - c_t - nK \\ \dot{S} &= -r_t \end{aligned}$$

Linear production function

$$F(K,r) = aK + br$$

$$D(t) = \frac{n}{(n-a)e^{nt}+a} \qquad Z(t) = \frac{1}{1+D(t)\frac{a}{n^2}t}$$

$$\Omega \equiv U + U_1'\left(\dot{k} - \xi(t,k,s)\right) + U_1'\dot{s}Db.$$

Cobb-Douglas production function

$$F(K,r) = K^\alpha r^\beta.$$

$$D(t) = e^{n(\alpha-\beta)t} \qquad Z(t) = -ne^{nt}$$

$$\Omega \equiv U + U_1'\left[\dot{k} + \eta\dot{s} - nk\right]$$

$$\eta = \beta e^{n(\alpha-\beta)t}k^\alpha r^{\beta-1}$$

The Dasgupta-Heal-Solow Model

$$\max_{c(.),r(.)} \int_0^\infty Z(t)U(c_t)dt$$

$$\begin{aligned}\dot{K} &= K_t^\alpha r_t^\beta - c_t \\ \dot{S} &= -r_t\end{aligned}$$

$$Z(t) = \frac{1}{v_1 t + w_1}$$

$$\Omega \equiv U + U_c'(\dot{K} + \dot{S}F_r') - U_c' Z(\xi + F_r'\mu)$$

Interpretation of the conservation law

Utility + change in stocks value

The net revenue of the economy (consumption + investment) is constant

Hartwick's Investment Rule $\Longrightarrow$ Constant utility

Conclusion

- We face the same restrictive conditions for the existence of economic conservation laws
- Conservation of the net revenue
- If sustainability requires something to be conserved, discounted utilitarian criterion may not be the best way to define what to be conserved

Some technical points

$$\max_{U(.)} \int_a^b L(X, \dot{X}, t)dt \qquad (1)$$

$$\dot{X} = f(X, U, t) \qquad (2)$$

The method

We search *transformations*

$$\begin{aligned} \bar{t} &= \phi(t, x, \varepsilon), & (3)\\ \bar{X}^i &= \psi^i(t, X, \varepsilon), \qquad (i = 1, \ldots, n). & (4) \end{aligned}$$

that satisfy the *Fundamental Invariance Identity*

$$\frac{\partial L}{\partial t}\tau + \sum_{i=1}^{n}\left(\frac{\partial L}{\partial x^i}\xi^i + \frac{\partial L}{\partial \dot{x}^i}\left(\frac{d\xi^i}{dt} - \dot{x}^i\frac{d\tau}{dt}\right)\right) + L\frac{d\tau}{dt} = 0. \qquad (5)$$

where τ and ξ^i, the first order coefficients of the Taylor series of ϕ and ψ^i around $\varepsilon = 0$, are the infinitesimal generators of the transformations.

$$\begin{aligned} \bar{t} &= t + \tau(t, k, s)\ \varepsilon \\ \bar{k} &= k + \xi(t, k, s)\ \varepsilon \\ \bar{s} &= s + \mu(t, k, s)\ \varepsilon \end{aligned}$$