Traffic grooming in a passive star WDM network.

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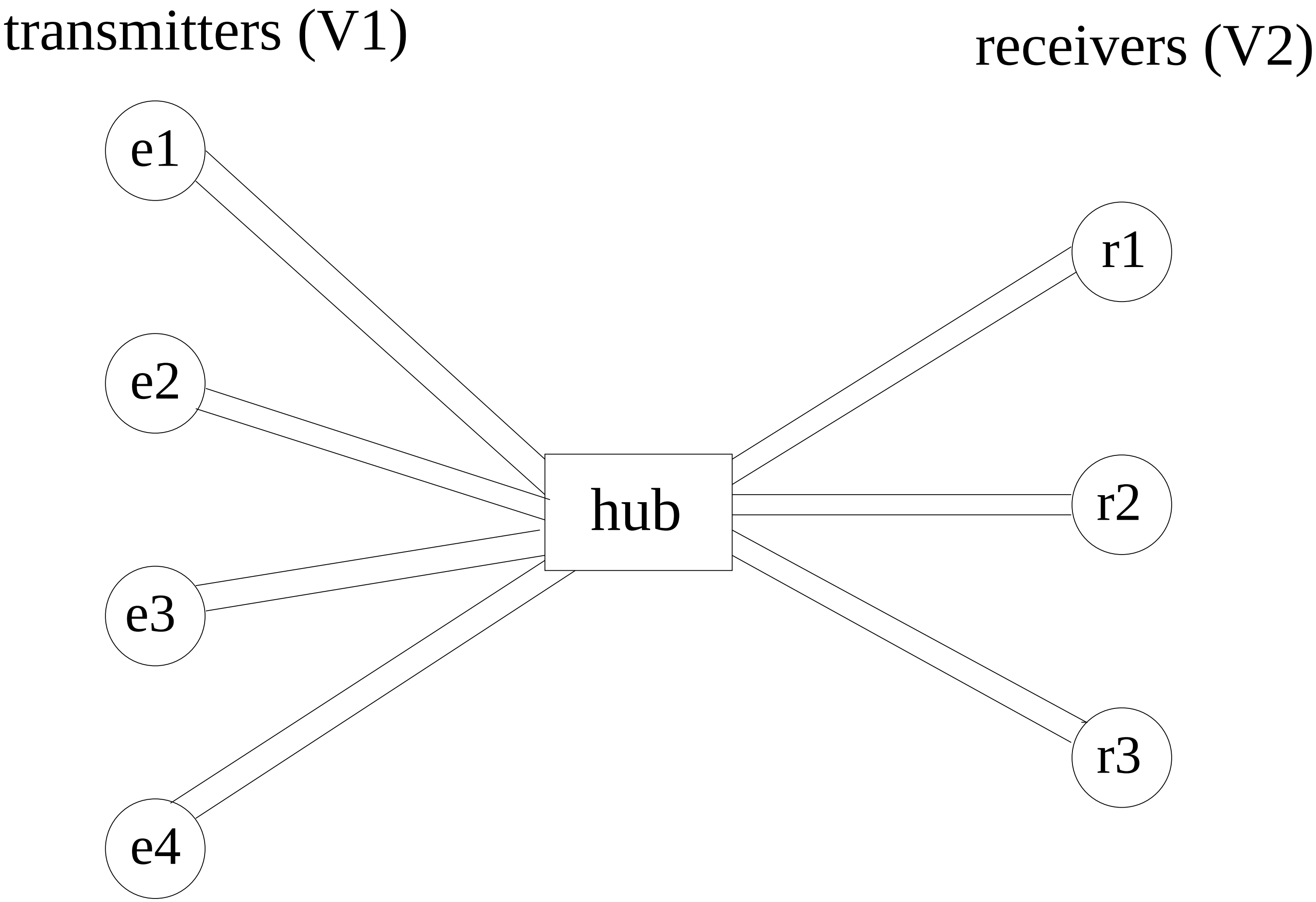
Outline

- Presentation of the problem
- State of the art
- A polynomial case
- Approximation algorithms
    - Maximization variant
    - Minimization variant
- Conclusion

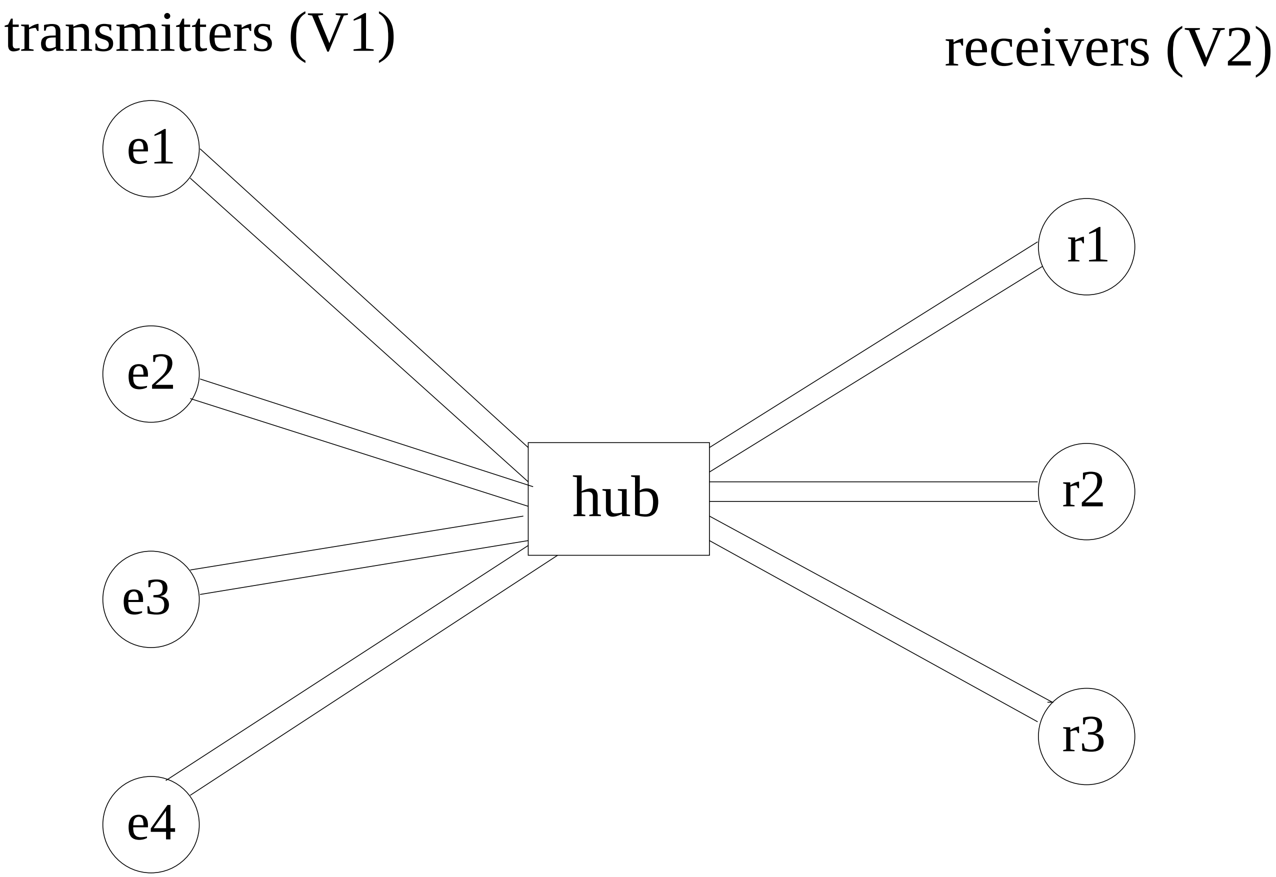
Presentation of the problem



W = number of links per fiber (= frequencies, = wavelengths)
C = capacity of a link

Presentation of the problem

Topology



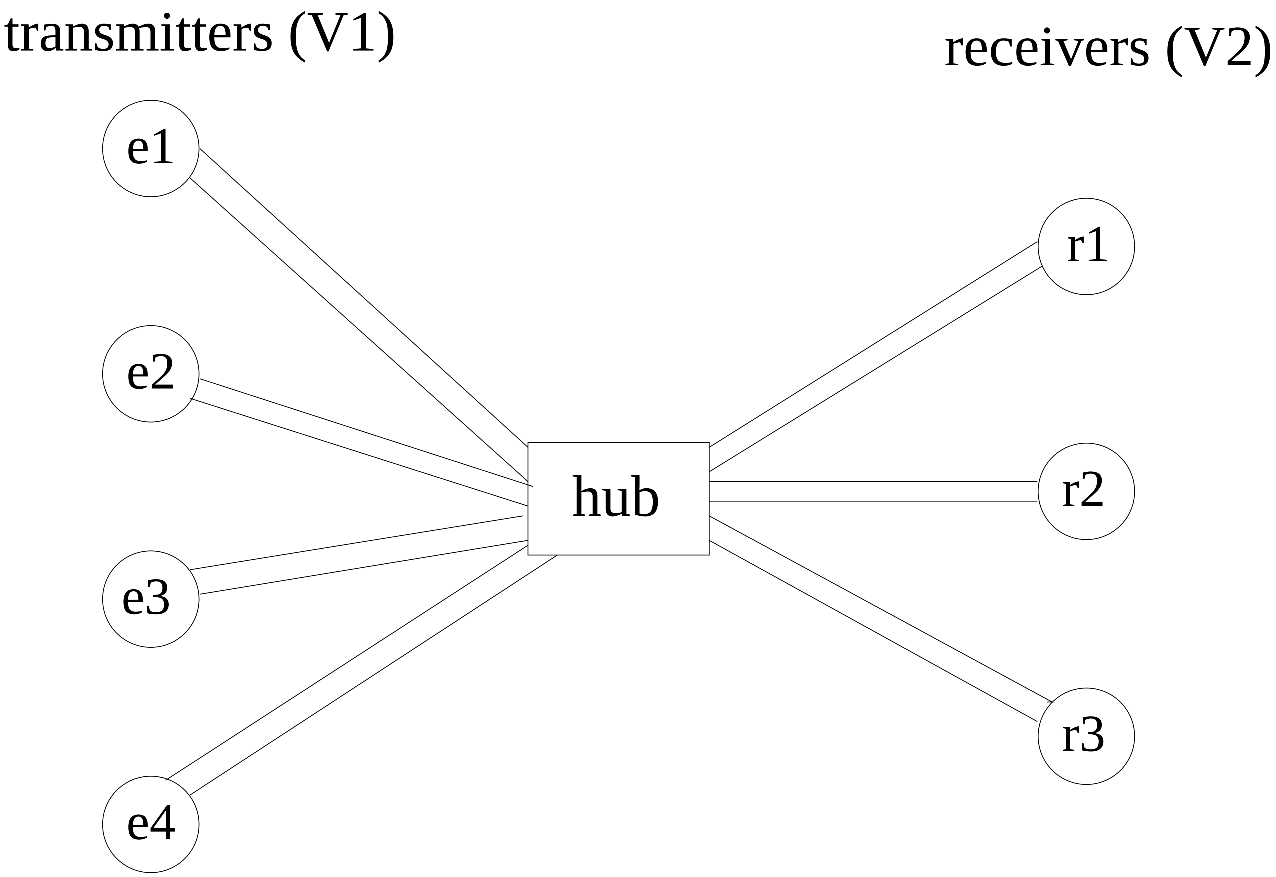
$W = 2,\ C = 10$

Traffic Matrix (requests)

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \\ 3 & 1 & 6 \end{pmatrix}$$

Presentation of the problem

Topology



$$W = 2,\ C = 10$$

Traffic Matrix (requests)

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \\ 3 & 1 & 6 \end{pmatrix}$$

All the requests can be routed iff:
$\forall i \in V_1, \quad \sum_{j \in V_2} t_{ij} \leq WC$, and $\forall j \in V_2, \quad \sum_{i \in V_1} t_{ij} \leq WC$.

Traffic grooming

A request may be routed

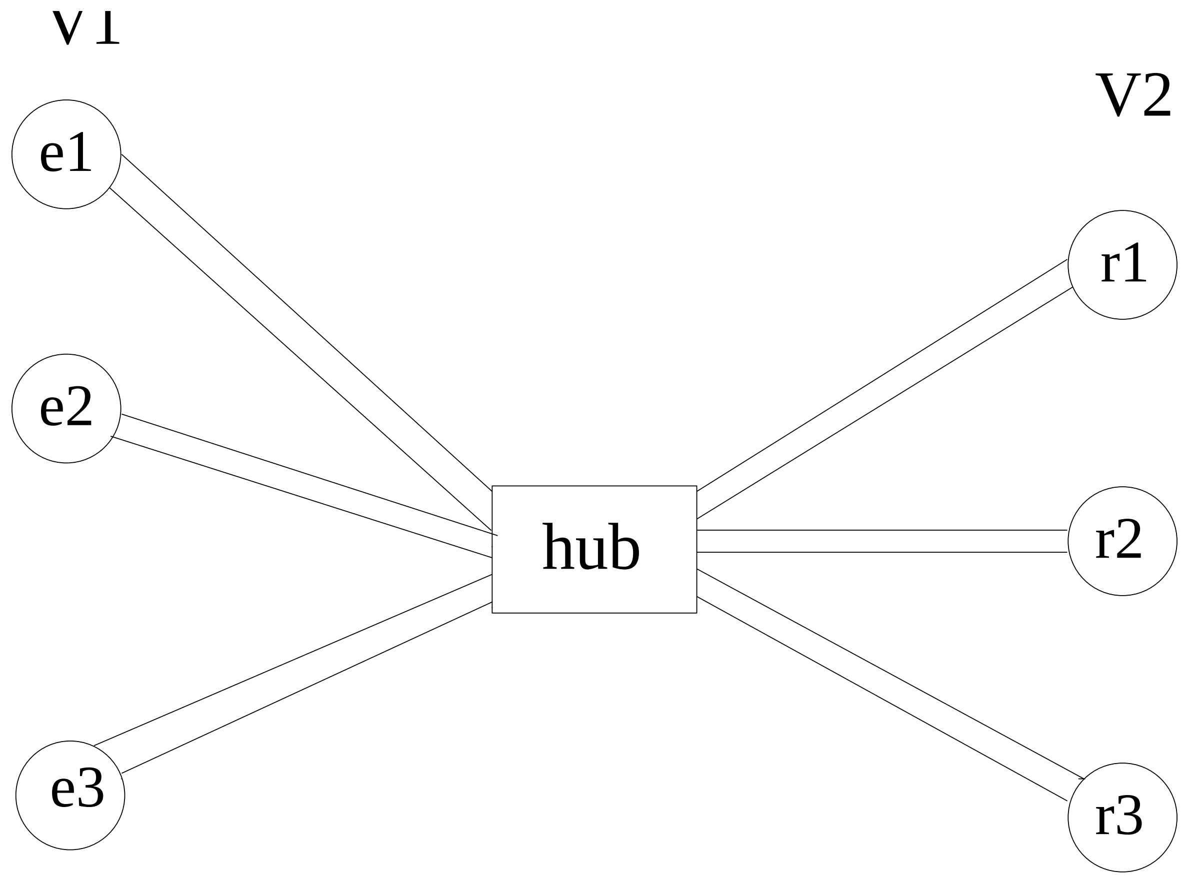
optically: it needs a frequency.

electronically: it may use the same frequency with other requests.

Traffic grooming

Example:

Topology



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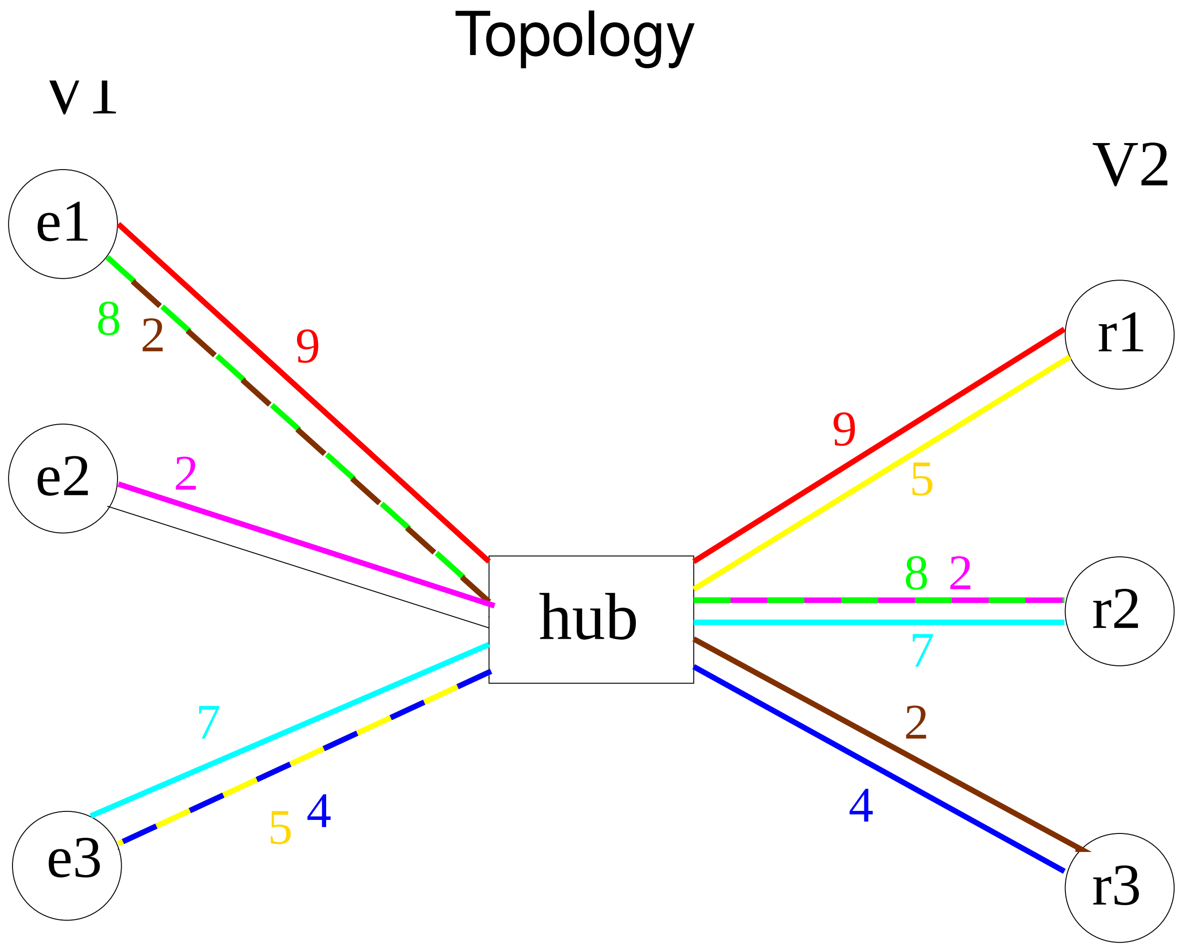
Traffic Matrix (requests)

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \end{pmatrix}$$

Example:



$W = 2,\ C = 10$

Traffic Matrix (requests)

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \end{pmatrix}$$

The problem

Input: Topology (C, W) + traffic matrix

Objective: Determine the set of optically (or electronically) routed requests

in such a way that:

the optically (resp. electronically) routed traffic is being maximized (minimized).

State of the art

R. Dutta, S. Huang, G. Rouskas: *Traffic grooming in Path, Star, and Tree Networks: Complexity, Bounds, and Algorithms,*
ACM SIGMETRICS 2003.

- different topologies (ring, star, tree)
- complexity results
- exact algorithms, heuristics with no performance guarantee

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Case where $W = 2$

Reduction to the problem of weighted maximum matching

Observation:

- In each row in which there are more than 2 values $\neq 0$, it is possible to route optically at most one request.
- When there are 2 values $\neq 0$ in a row, then we may possibly route optically 2 requests.

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \end{pmatrix} \quad W = 2,\ C = 10$$

Case where $W = 2$

The reduction

Let $\mathbf{T}' = \mathbf{T}$ in which we duplicate the rows (and columns) having only 2 values $\neq 0$.

In each row (column) of $\mathbf{T}'$, we may route optically at most one request.

$$\mathbf{T} = \begin{pmatrix} 9 & 8 & 2 \\ 0 & 2 & 0 \\ 5 & 7 & 4 \end{pmatrix} \qquad \mathbf{T}' = \begin{pmatrix} 9 & 0 & 8 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 5 & 7 & 0 & 4 \end{pmatrix}$$

Case where $W = 2$

We transform $\mathbf{T}'$ to $\mathbf{M}$:

$$m_{ij} = \begin{cases} t_{ij} & \text{if the request from i to j can be optically routed.} \\ -\infty & \text{otherwise.} \end{cases}$$

$$\mathbf{T}' = \begin{pmatrix} 9 & 0 & 8 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 5 & 7 & 0 & 4 \end{pmatrix} \quad \mathbf{M} = \begin{pmatrix} 9 & -\infty & -\infty & -\infty & -\infty \\ 0 & 0 & 2 & 0 & 0 \\ 0 & -\infty & 7 & 0 & -\infty \end{pmatrix}$$

We search a maximum weighted matching in M (we choose at most 1 request per row, and at most 1 per column).

$\rightarrow$ we get the requests that have to be routed optically in an optimal solution.

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Maximization

Reduction to a b-matching problem

Integer Linear Program

$$\text{Maximize} \sum_{i \in V_1, j \in V_2} y_{ij} t_{ij} \tag{1}$$

subject to:

$$\forall i \in V_1, \sum_{j \in V_2} y_{ij}(C - t_{ij}) \leq CW - \sum_{j \in V_2} t_{ij} \tag{2}$$

$$\forall j \in V_2, \sum_{i \in V_1} y_{ij}(C - t_{ij}) \leq CW - \sum_{i \in V_1} t_{ij} \tag{3}$$

$$\forall i \in V_1, j \in V_2, y_{ij} \in \{0, 1\} \tag{4}$$

$y_{ij} = 1 \Rightarrow t_{ij}$ is optically routed.
$y_{ij} = 0 \Rightarrow t_{ij}$ is electronically routed.

The b-matching problem:

Given:

$G = (V, E)$

Each vertex v has a capacity b_v.

Each edge e has: - a demand d_e
- a profit p_e.

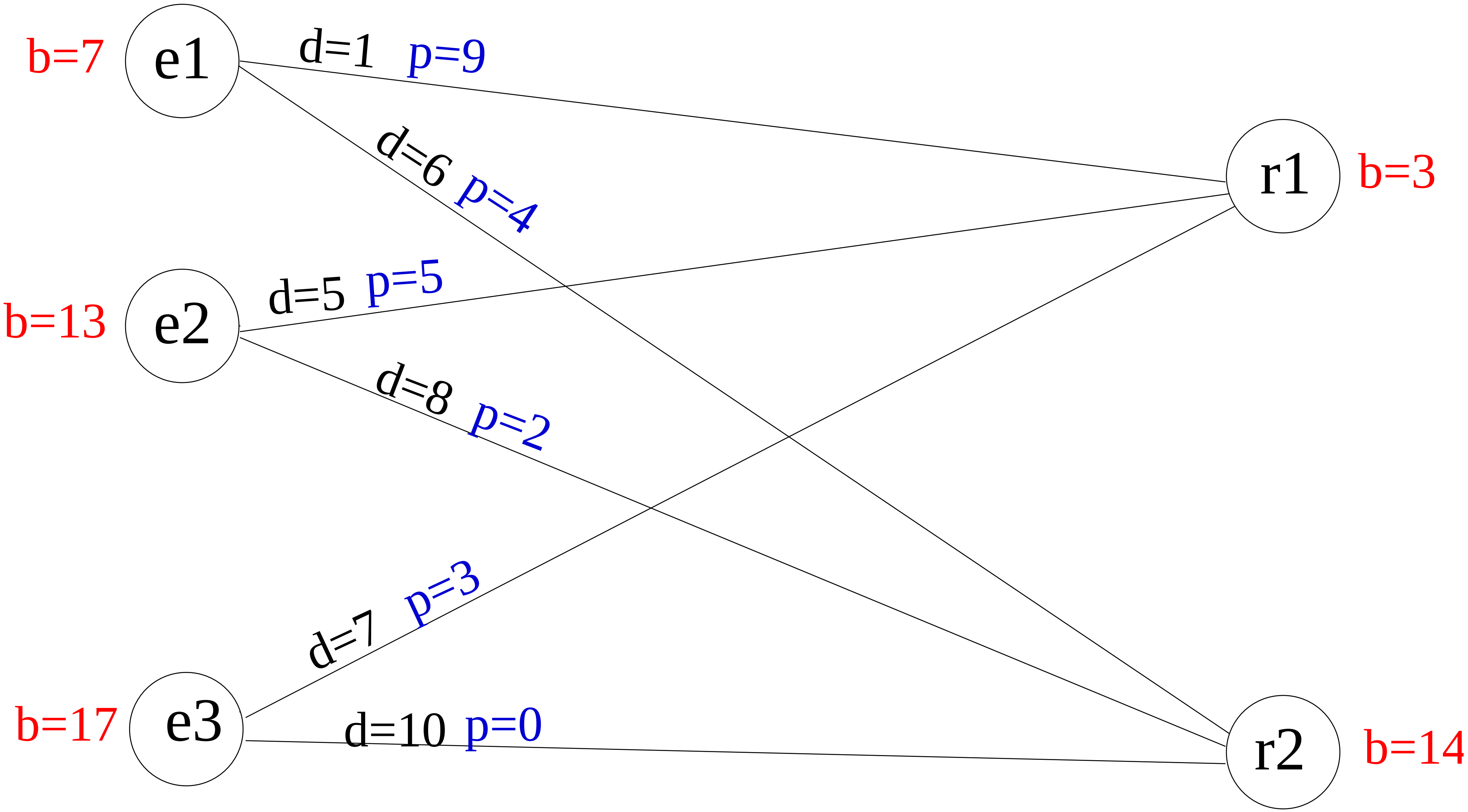
We search:

A subset M of edges s.t. $\forall v \in V, \sum_{e \in \delta(v) \cap M} d_e \leq b_v$.

We want to maximize $\sum_{e \in M} p_e$.

Maximization

Example:



Maximization

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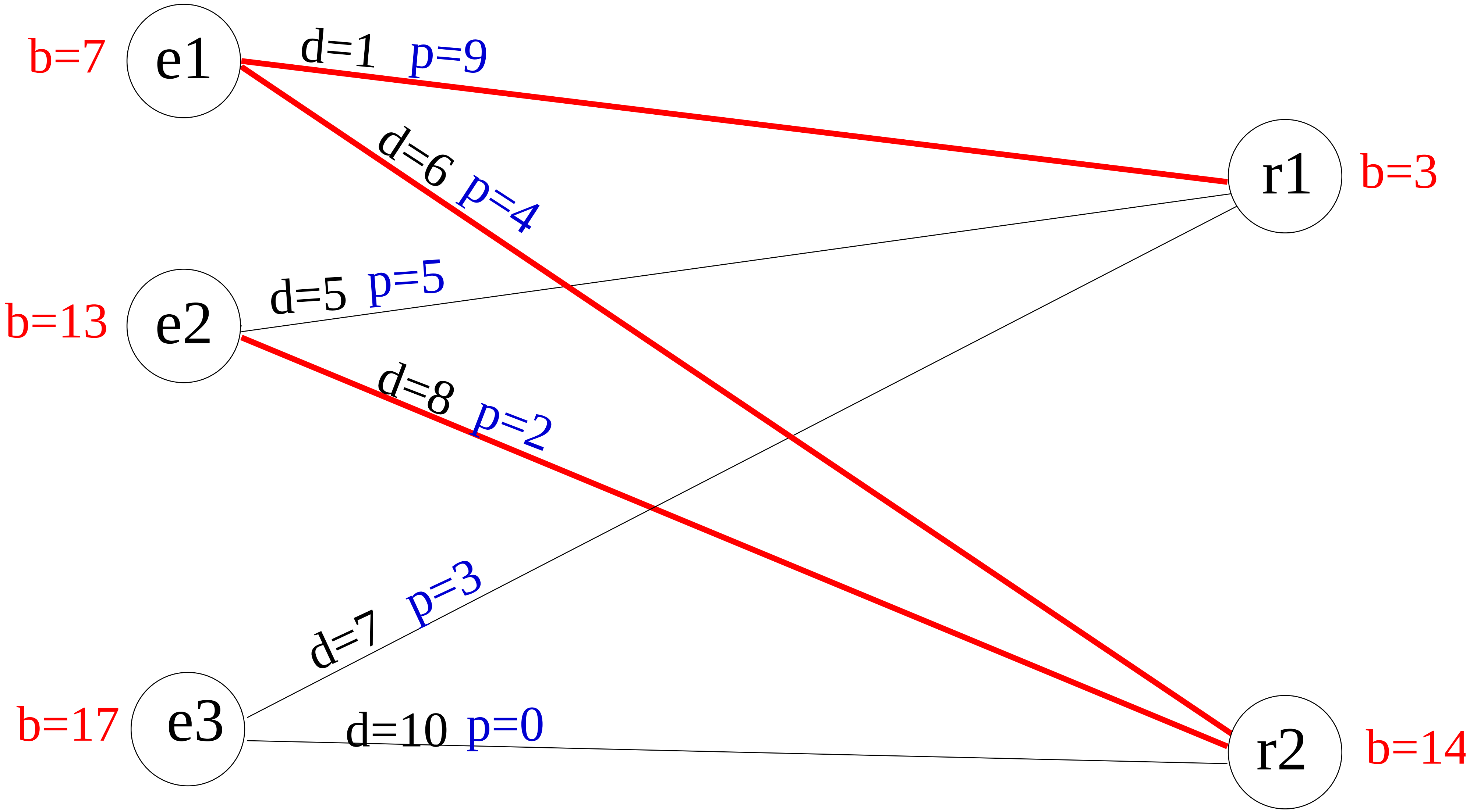


Maximization

Theorem: [Sepherd, Vetta 2004]
There is a polynomial time 2.8-approximation algorithm for the b-matching problem in a bipartite graph.

Theorem:
There is a polynomial time 2.8-approximation algorithm for the maximization version of the traffic grooming problem in a passive star.

Proof:
We transform the problem into a b-matching problem in a bipartite graph.

Maximization

b-matching problem:

Given:
$G = (V, E)$
Each vertex v has a capacity b_v.
Each edge e has: - a demand d_e
- a profit p_e.

We search:
A subset M of edges s.t.
$\forall v \in V, \sum_{e \in \delta(v) \cap M} d_e \leq b_v$.
We want to maximize $\sum_{e \in M} p_e$.

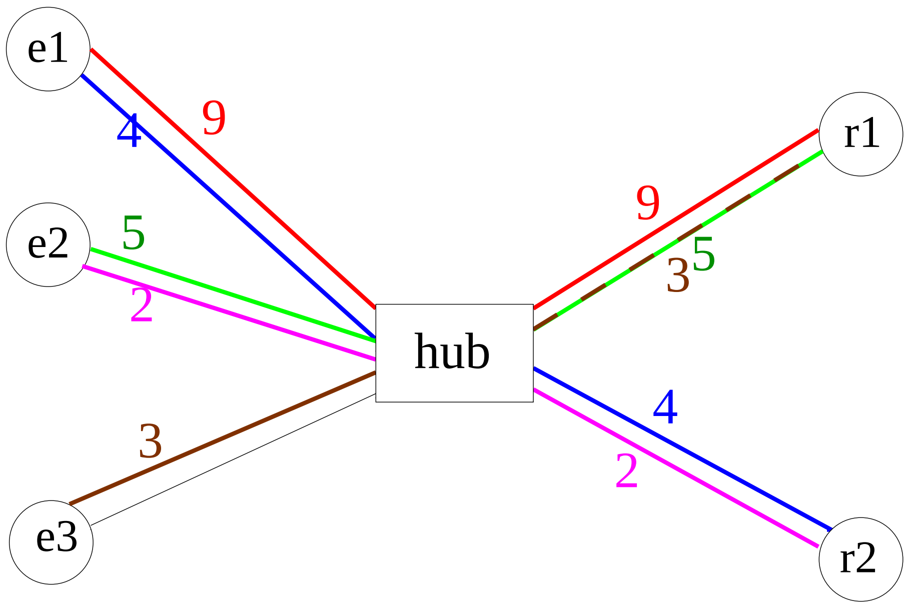
Transformation:

$\forall v \in V_1,\ b_v = CW - \sum_{j \in V_2} t_{vj}$
$\forall v \in V_2,\ b_v = CW - \sum_{i \in V_1} t_{iv}$

$\forall e \in (V_1 \times V_2),\ d_e = C - t_e$
$\forall e \in (V_1 \times V_2),\ p_e = t_e$

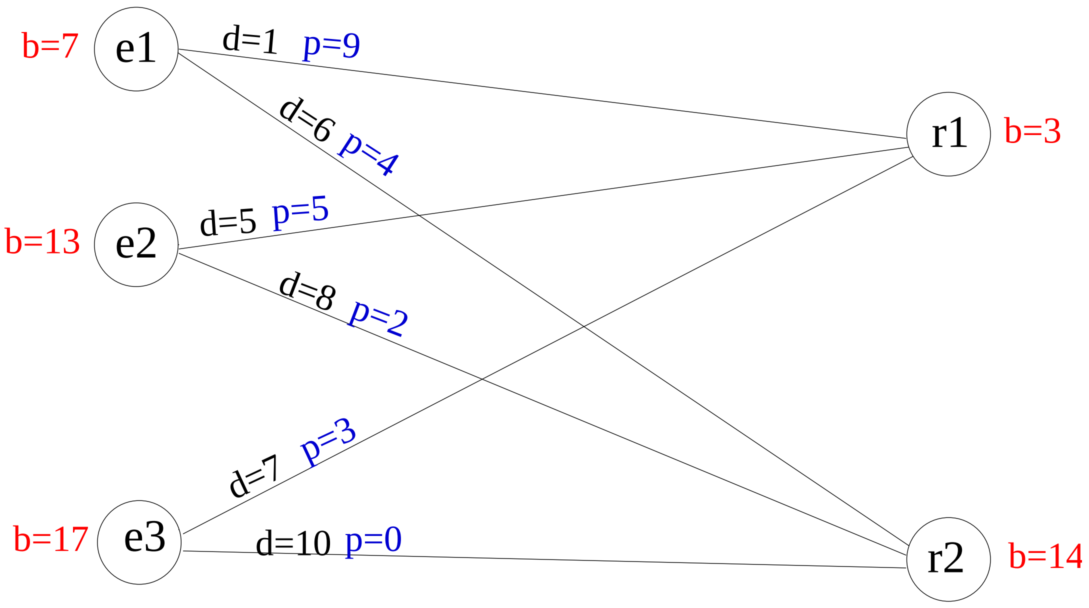
Maximization

Traffic grooming



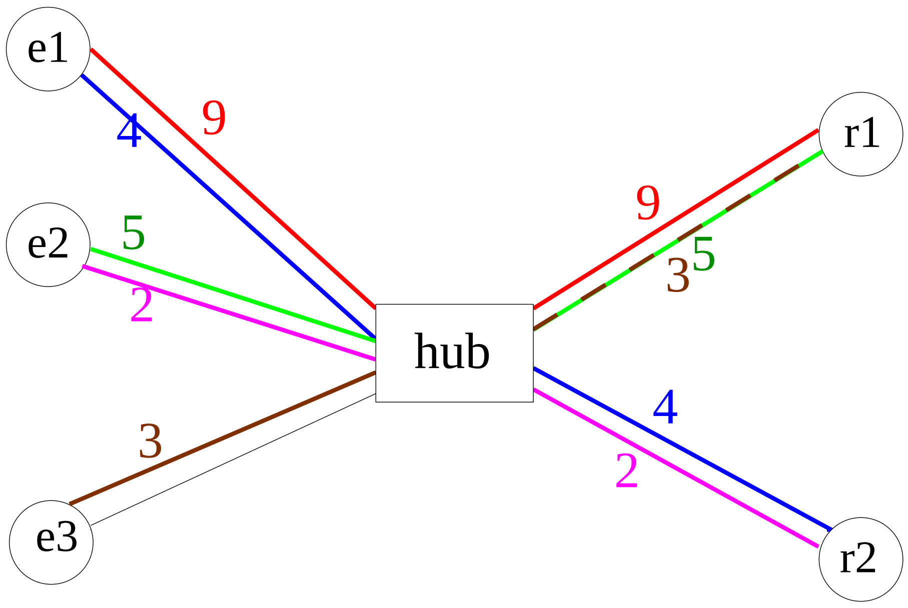
$$W = 2, C = 10$$

b-matching



Maximization

Traffic grooming



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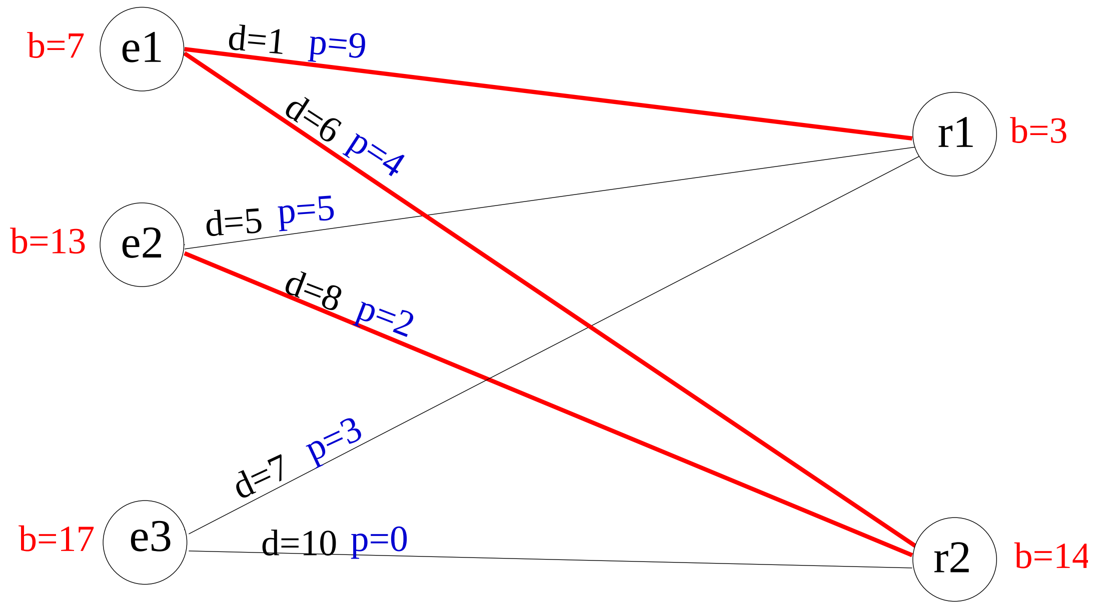
b-matching



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Minimization

Reduction to the constrained multiset multicover problem

Integer Linear Program:

$$\text{Minimize} \sum_{i \in V_1, j \in V_2} x_{ij} t_{ij} \tag{5}$$

subject to:

$$\forall i \in V_1, \sum_{j \in V_2} x_{ij}(C - t_{ij}) \geq C(|V_2| - W) \tag{6}$$

$$\forall j \in V_2, \sum_{i \in V_1} x_{ij}(C - t_{ij}) \geq C(|V_1| - W) \tag{7}$$

$$\forall i \in V_1, j \in V_2, x_{ij} \in \{0, 1\} \tag{8}$$

$x_{ij} = 1 \Rightarrow t_{ij}$ is electronically routed.
$x_{ij} = 0 \Rightarrow t_{ij}$ is optically routed.

Minimization

Theorem: [Rajagopalan, Vazirani 1999]
There is a polynomial time H_{2C}-approximation algorithm for the constrained multiset multicover problem.

$$(H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n})$$

Theorem:
There is a polynomial time H_{2C}-approximation algorithm for the minimization version of the traffic grooming problem.

Proof:
We transform the problem into the constrained multiset multicover problem.

Conclusion and Future work

Conclusion:

We have:

- A polynomial algorithm for the case where $W = 2$.
- Two approximation algorithms (2.8 et H_{2C}).

Conclusion and Future work

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We have:

- A polynomial algorithm for the case where $W = 2$.
- Two approximation algorithms (2.8 et H_{2C}).

Future work:

- Compare the two algorithms.
- Propose an ad-hoc algorithm for the problem
- Study the (non)-approximability of the traffic grooming problem for other (more realistic) topologies.

Minimization

Theorem: [Rajagopalan, Vazirani 1999]
There is a polynomial time H_{2C}-approximation algorithm for the constrained multiset multicover problem.

Theorem:
There is a polynomial time H_{2C}-approximation algorithm for the minimization version of the traffic grooming problem in a passive star.

Proof:
We transform the problem to the constrained multiset multicover problem.

Constrained multiset multicover problem:
Given: A set of elements E, each element e has to be covered n_e times. A collection of multisets S, each multiset having a cost.
Objective: We have to choose a minimum cost subset of S in such a way that each element e of E is covered at least n_e times.

Minimization

$$\text{Minimize} \sum_{i \in V_1, j \in V_2} x_{ij} t_{ij}$$

subject to:

$$\forall i \in V_1, \sum_{j \in V_2} x_{ij}(C - t_{ij}) \geq C(|V_2| - W)$$

$$\forall j \in V_2, \sum_{i \in V_1} x_{ij}(C - t_{ij}) \geq C(|V_1| - W)$$

$$\forall i \in V_1, j \in V_2, x_{ij} \in \{0, 1\}$$

Transformation:

- Elements to be covered: $e_i (i \in V_1)$, $r_j (j \in V_2)$
- $\forall t_{ij}$, we create $S_{ij} = (\underbrace{e_i, \ldots, e_i}_{C - t_{ij}}, \underbrace{r_j, \ldots, r_j}_{C - t_{ij}})$ of cost t_{ij}.
- We have to cover
  e_i $C(|V_2| - W)$ times,
  r_j $C(|V_1| - W)$ times.

Minimization

Example: Let $W = 2, C = 5$, and $\mathbf{T} = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 3 & 4 \\ 2 & 0 & 5 \end{pmatrix}$

We obtain the following multisets:
$S_{1,1} = \{e_1, r_1\}$ and $c(S_{1,1}) = 4$
$S_{1,2} = \{e_1, e_1, e_1, r_2, r_2, r_2\}$ and $c(S_{1,2}) = 2$
$S_{1,3} = \{e_1, e_1, e_1, e_1, r_3, r_3, r_3, r_3\}$ and $c(S_{1,3}) = 1$
$S_{2,1} = \{e_2, e_2, r_1, r_1\}$ and $c(S_{2,1}) = 3$
$S_{2,2} = \{e_2, e_2, r_2, r_2\}$ and $c(S_{2,2}) = 3$
$S_{2,3} = \{e_2, r_3\}$ and $c(S_{2,3}) = 4$
$S_{3,1} = \{e_3, e_3, e_3, r_1, r_1, r_1\}$ and $c(S_{3,1}) = 2$
$S_{3,2} = \{e_3, e_3, e_3, e_3, e_3, r_2, r_2, r_2, r_2, r_2\}$ and $c(S_{3,2}) = 0$
$S_{3,3} = \{\emptyset\}$ and $c(S_{3,3}) = 5$
A solution: $\mathcal{C} = \{S_{1,2}, S_{1,3}, S_{2,1}, S_{2,2}, S_{2,3}, S_{3,1}, S_{3,2}\}$. Requests which will be electronically switched at the hub are then $T = \{t_{1,2}, t_{1,3}, t_{2,1}, t_{2,2}, t_{2,3}, t_{3,1}, t_{3,2}\}$, the others ($t_{1,1}$ and $t_{3,3}$) will be optically routed.