Study of New Semantics for Regular Path Queries

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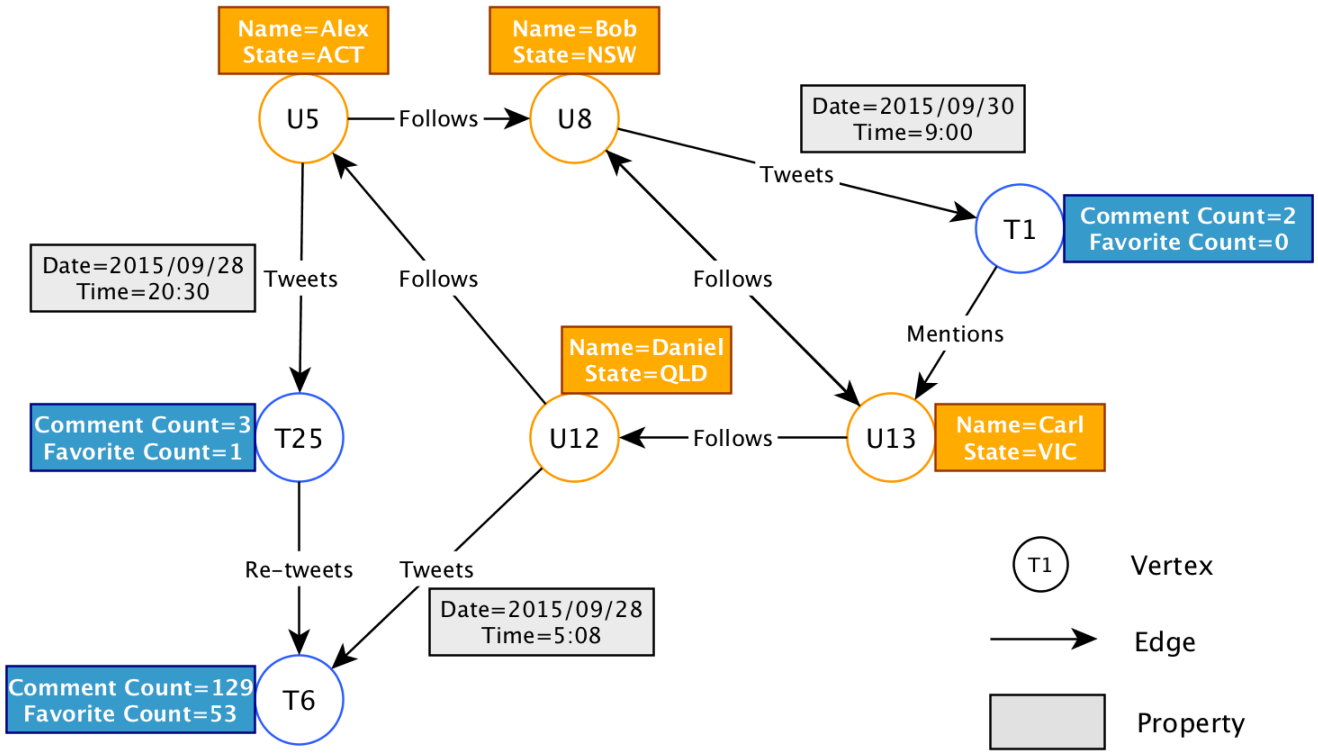
[1]Supervised by Amélie Gheerbrant, Victor Marsault and Antoine Meyer.

Introduction

Graph Database

In practice, we have property graphs.



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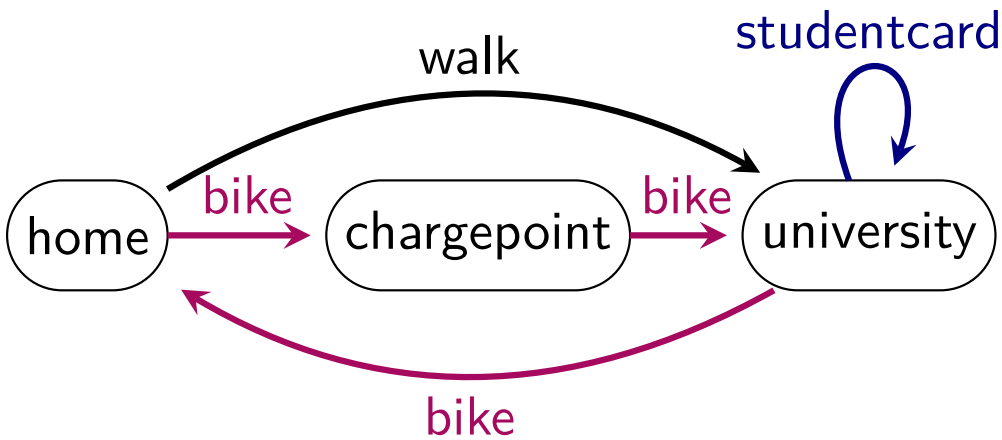
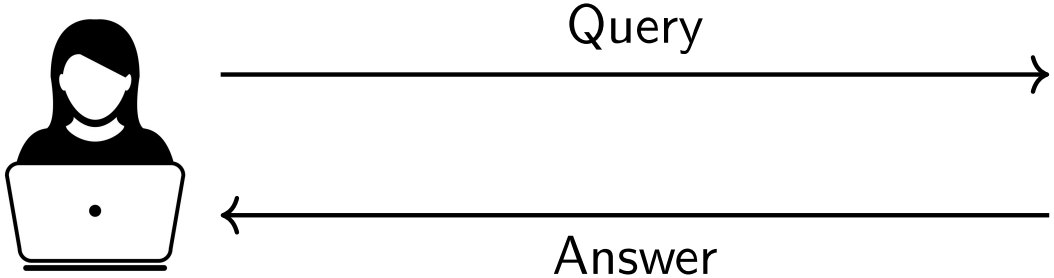
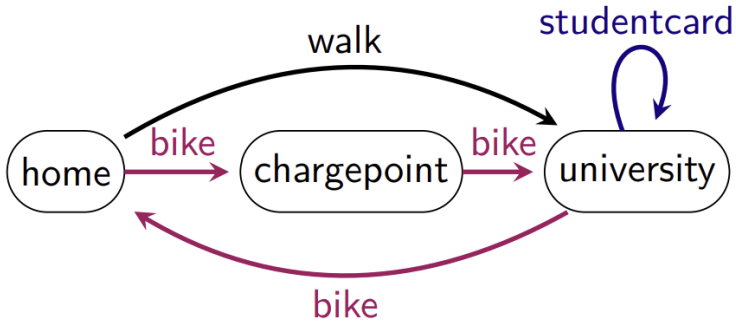
Our abstraction

Directed, multi-labeled, multi-edge graph.



Regular Path Queries





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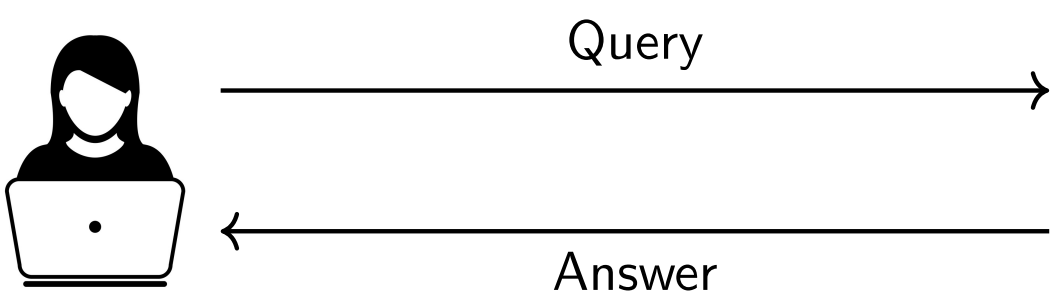

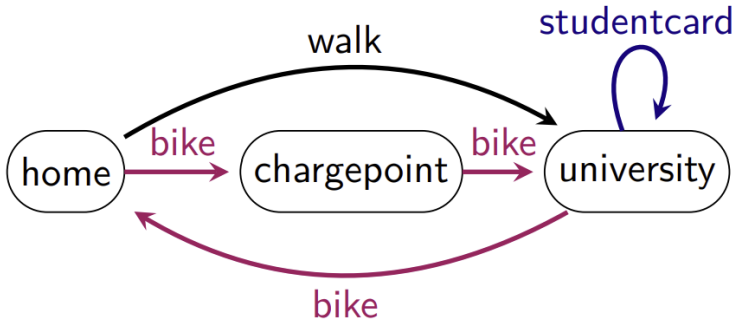

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Represented by a regular expression R or an automaton A.

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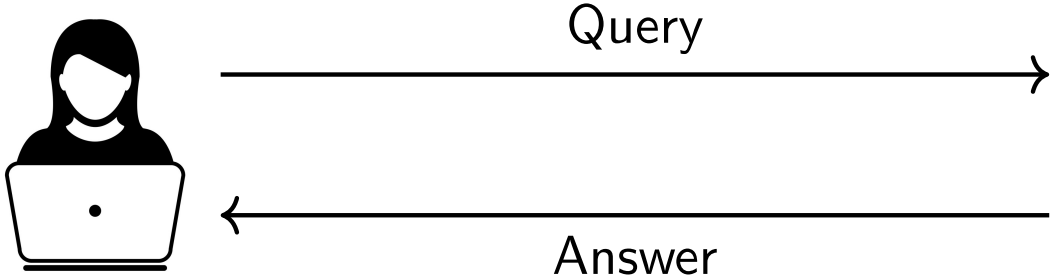

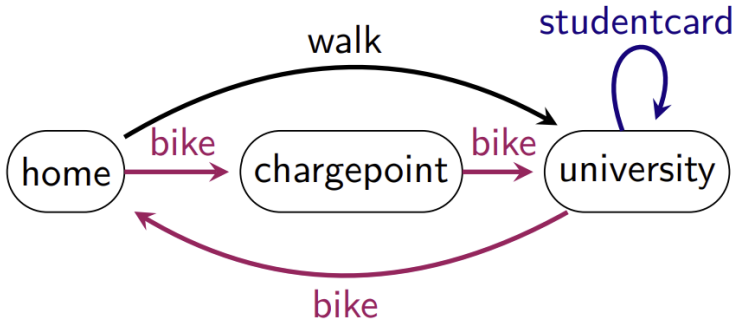

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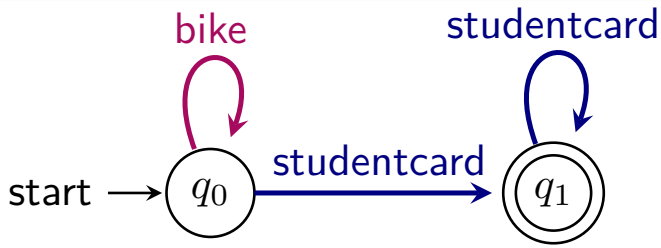
Query: From home to university

$$R = \text{bike}^* \cdot \text{studentcard}^+$$



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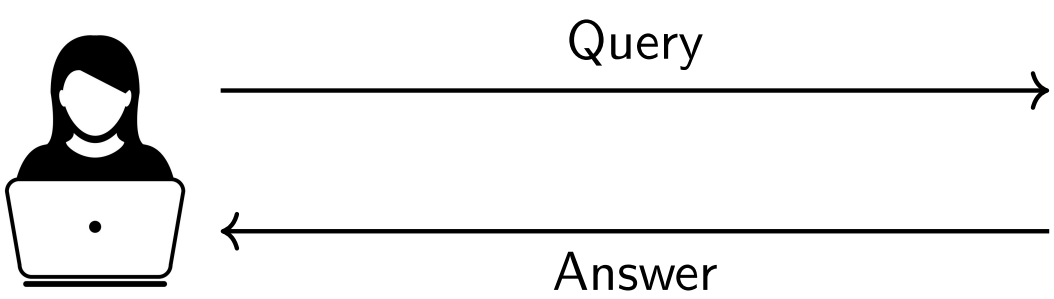

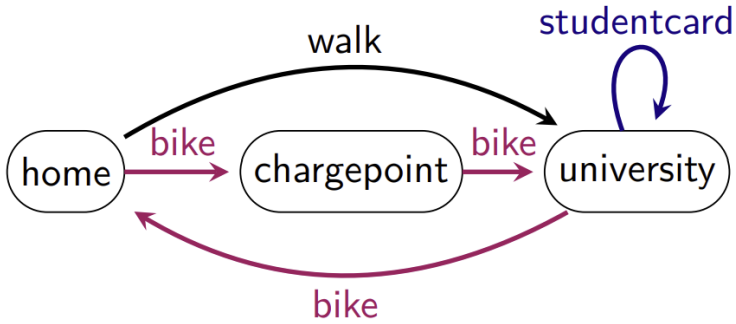

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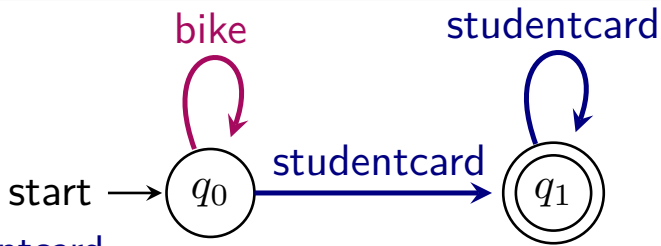

Answer: home $\xrightarrow{\text{bike}}$ chargepoint $\xrightarrow{\text{bike}}$ university $\xrightarrow{\text{studentcard}}$ university

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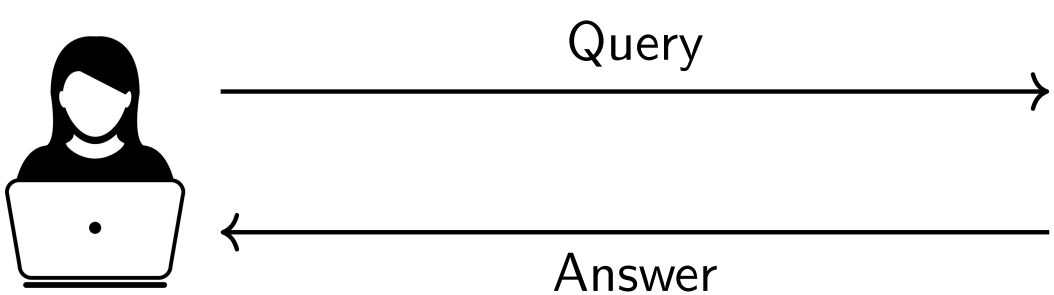

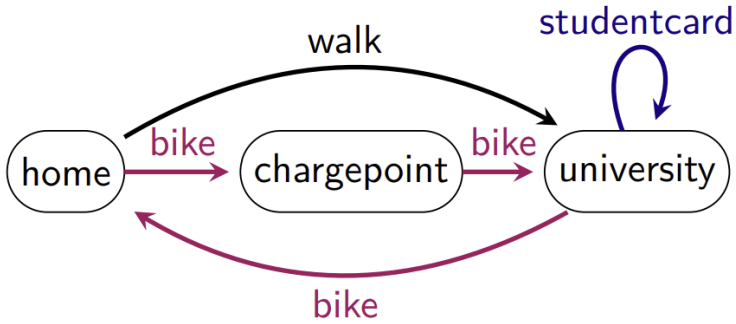

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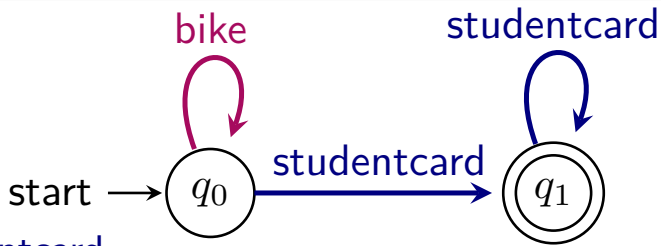

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Walk

Alternating sequence of vertices and edges.

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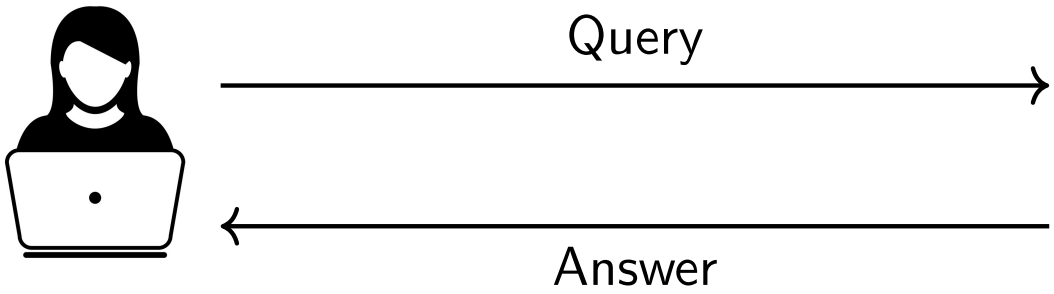

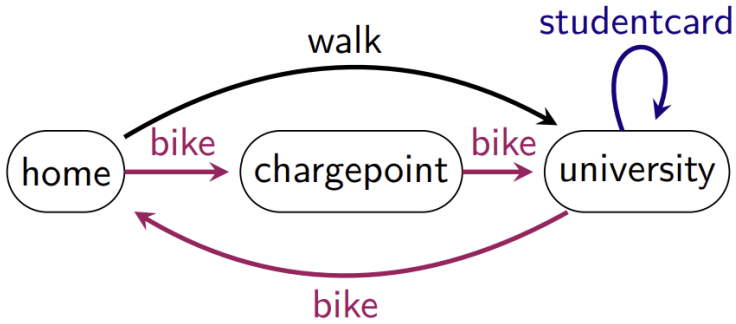

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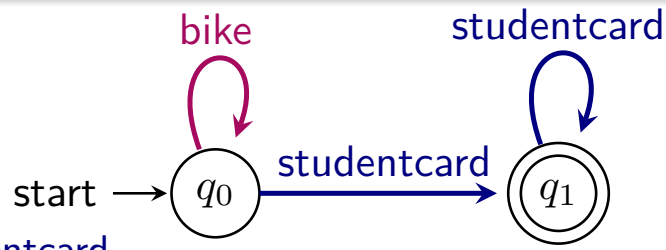

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$$\textsc{Match}_{\mathcal{A}}(D)$$

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$D \times A$

Product of the graph database D and the automaton A representing the RPQ.

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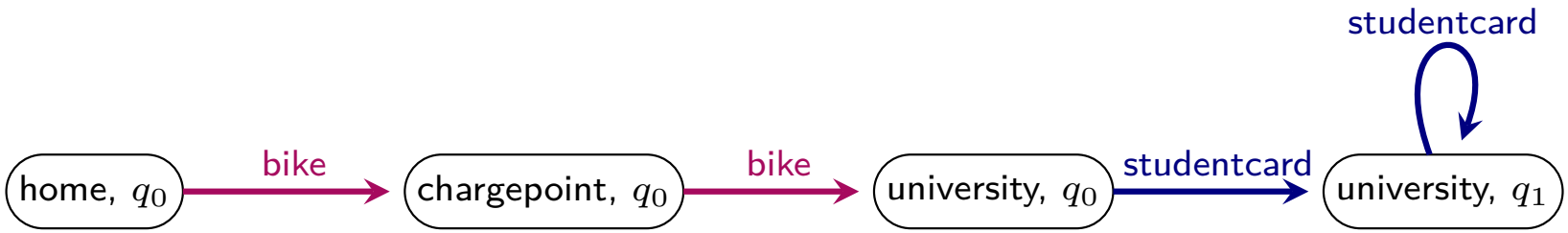

Figure: Part of the Product Graph

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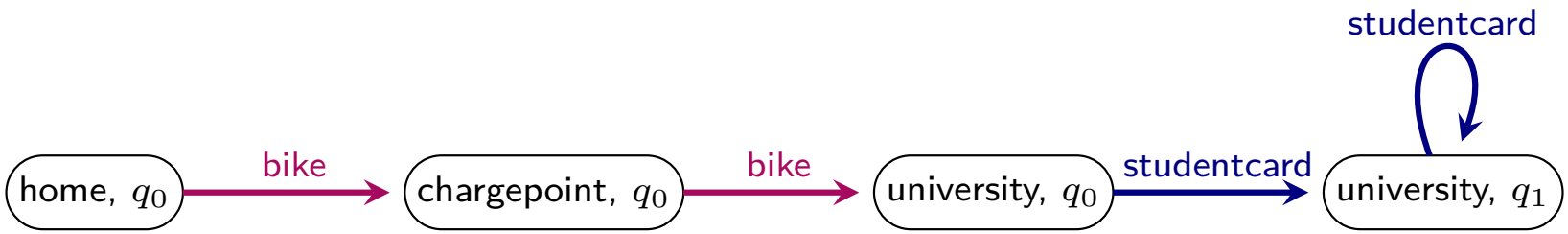

Figure: Part of the Product Graph

- **Run:** walk in the product graph $D \times A$.
  (home, q_0) $\xrightarrow{\text{bike}}$ (chargepoint, q_0) $\xrightarrow{\text{bike}}$ (university, q_0) $\xrightarrow{\text{studentcard}}$ (university, q_1)

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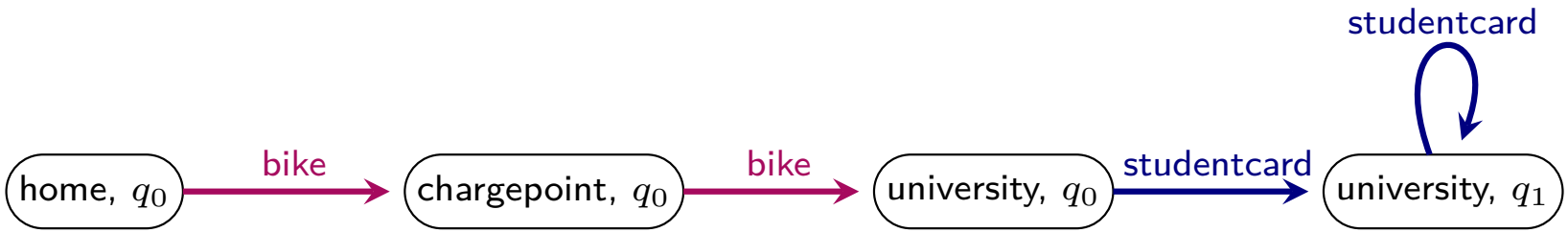

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- **Projection:** A walk $\in \text{MATCH}_{\mathcal{A}}(D)$.
  home $\xrightarrow{\text{bike}}$ chargepoint $\xrightarrow{\text{bike}}$ university $\xrightarrow{\text{studentcard}}$ university

Matches

Query: From home to university

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Issue? Infinite matches!

RPQ Semantics

Compute a finite and meaningful output from the matches.

Output of a semantics X: $[\![\mathcal{A}]\!]_X(D)$

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Criteria

- Coverage: Not defined in literature.
- Computational Properties: Complexity of the most common computational problems in database querying.

Motivations

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We need new candidates.

New RPQ Semantics

Shortest-coverage Semantics

Output

$$[\![\mathcal{A}]\!]_{SC}(D) = \bigcup_{(s,t,v)\in V^3} \{\!\{w \in \textsc{Match}_{\mathcal{A}}(D,s,t) \mid w \text{ is a shortest walk covering } v \text{ in } \textsc{Match}_{\mathcal{A}}(D,s,t)\}\!\}$$

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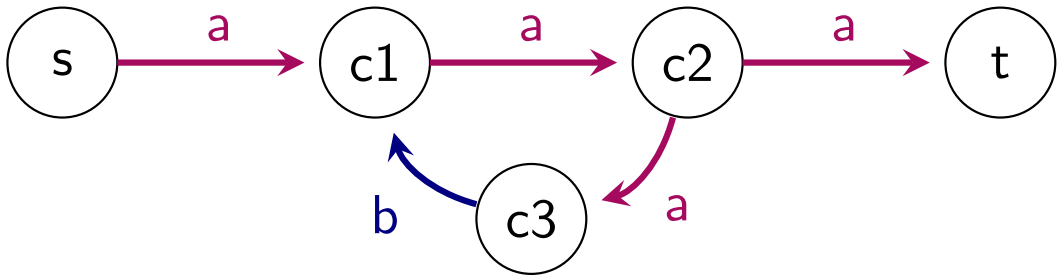

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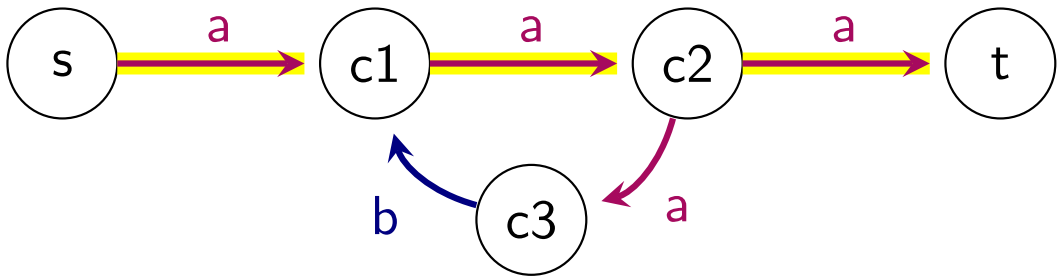

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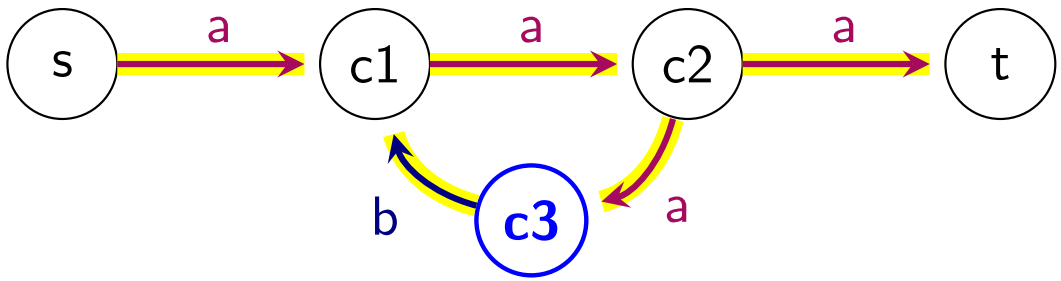

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- For s, c1, c2, t: s $\xrightarrow{a}$ c1 $\xrightarrow{a}$ c2 $\xrightarrow{a}$ t
- **For c3:** s $\xrightarrow{a}$ c1 $\xrightarrow{a}$ c2 $\xrightarrow{a}$ c3 $\xrightarrow{b}$ c1 $\xrightarrow{a}$ c2 $\xrightarrow{a}$ t

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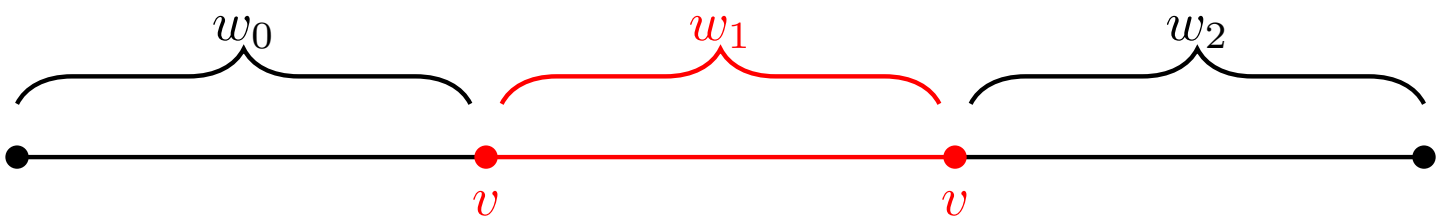

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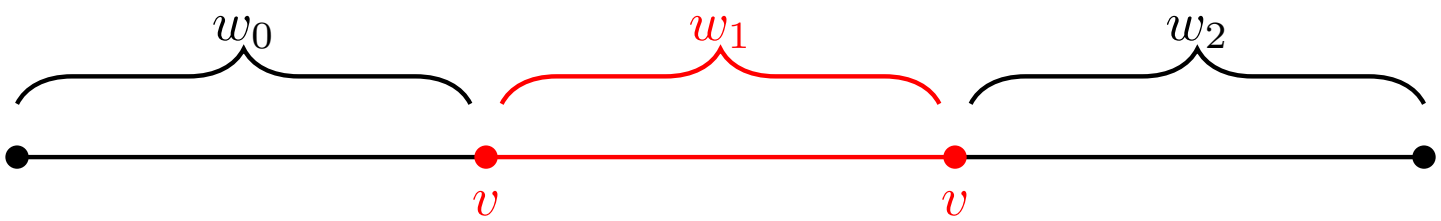

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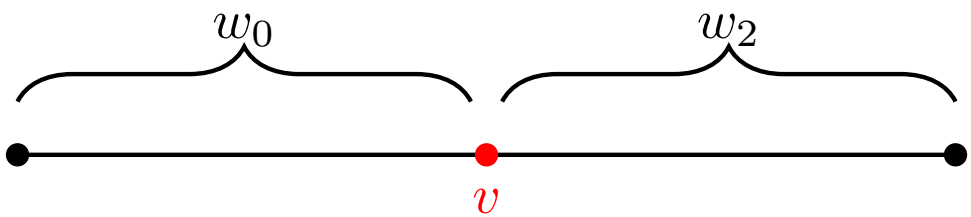

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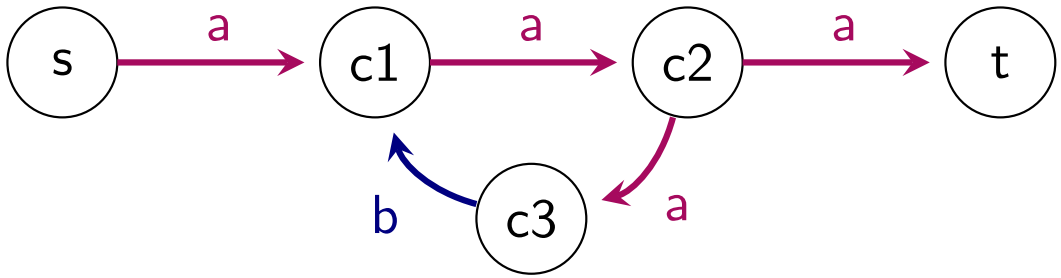

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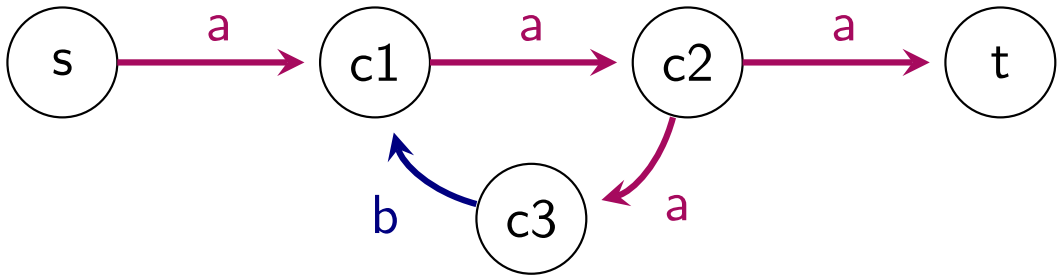

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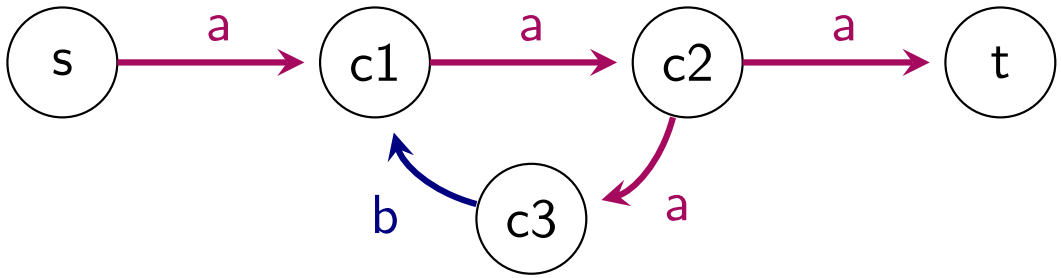

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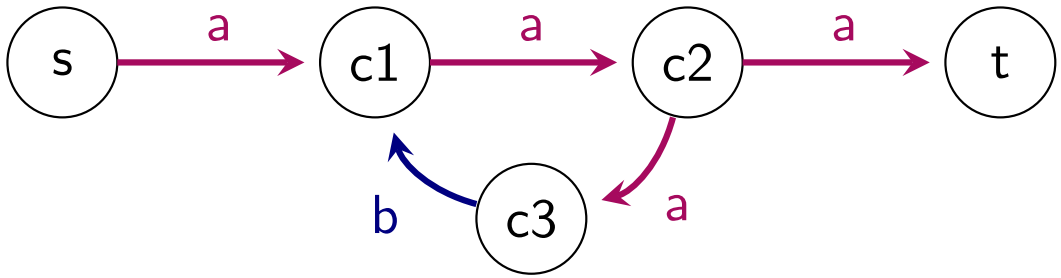

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Not a minimal walk! It has a subwalk that is a match.

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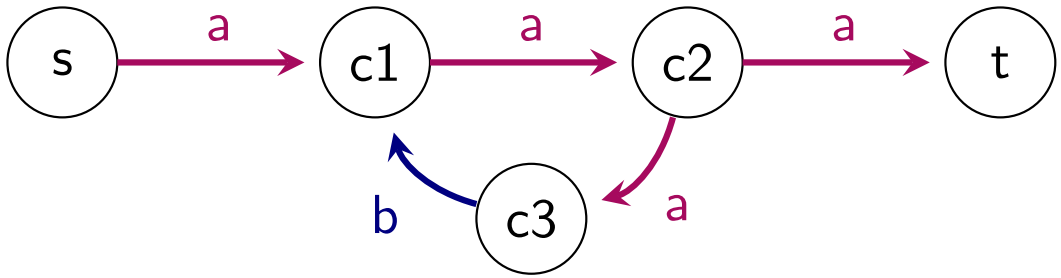

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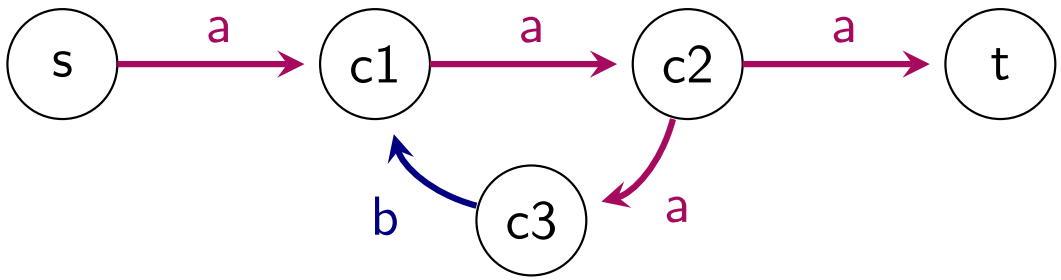

Query: From s to t $\qquad R = a^* b\ a^*$

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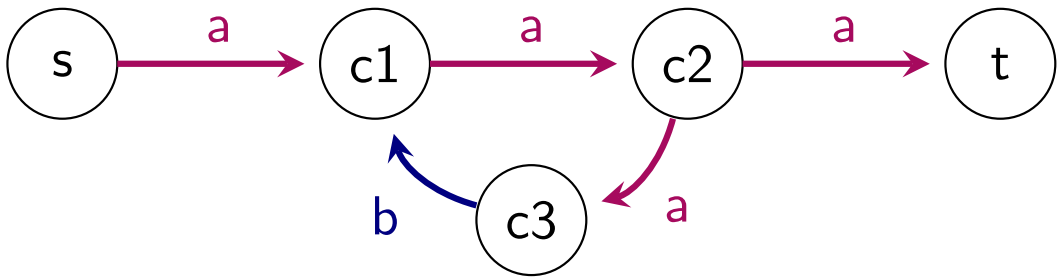

Query: From s to t $\quad R = a^* b\ a^*$

w: s $\xrightarrow{a}$ c1 $\xrightarrow{a}$ c2 $\xrightarrow{a}$ c3 $\xrightarrow{b}$ c1 $\xrightarrow{a}$ c2 $\xrightarrow{a}$ t

It is a minimal walk.

Minimal-walk Semantics

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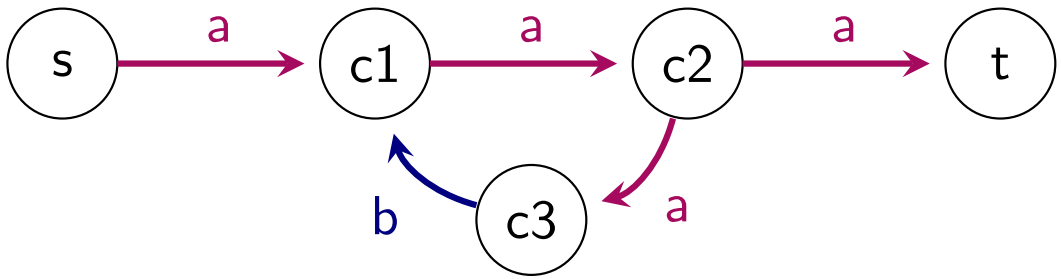

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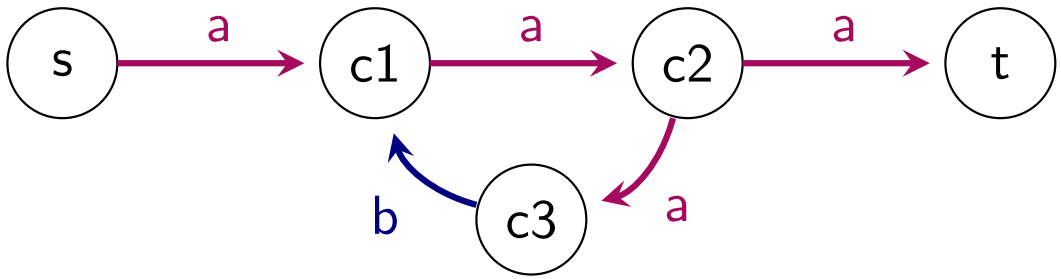

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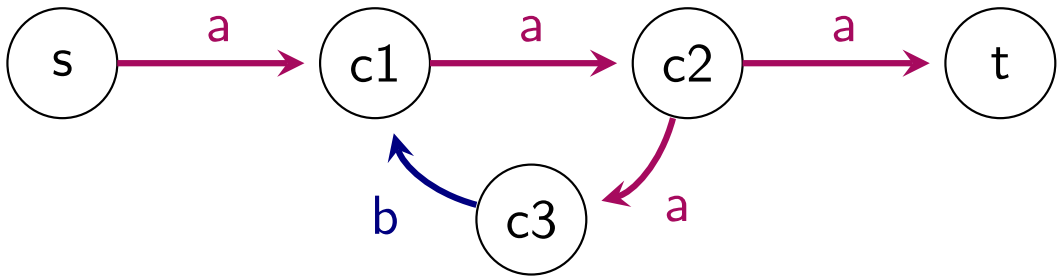

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Computational Problems: Walk Membership

Problem definition

Walk Membership Under X Semantics

Input: A graph database D, a query $\mathcal{A}$, a walk $w \in \text{Match}_{\mathcal{A}}(D)$.
Output: $w \in [\![\mathcal{A}]\!]_X(D)$?

Walk Membership under shortest-coverage semantics

Theorem

Walk Membership *under shortest-coverage semantics is in* ***P***.

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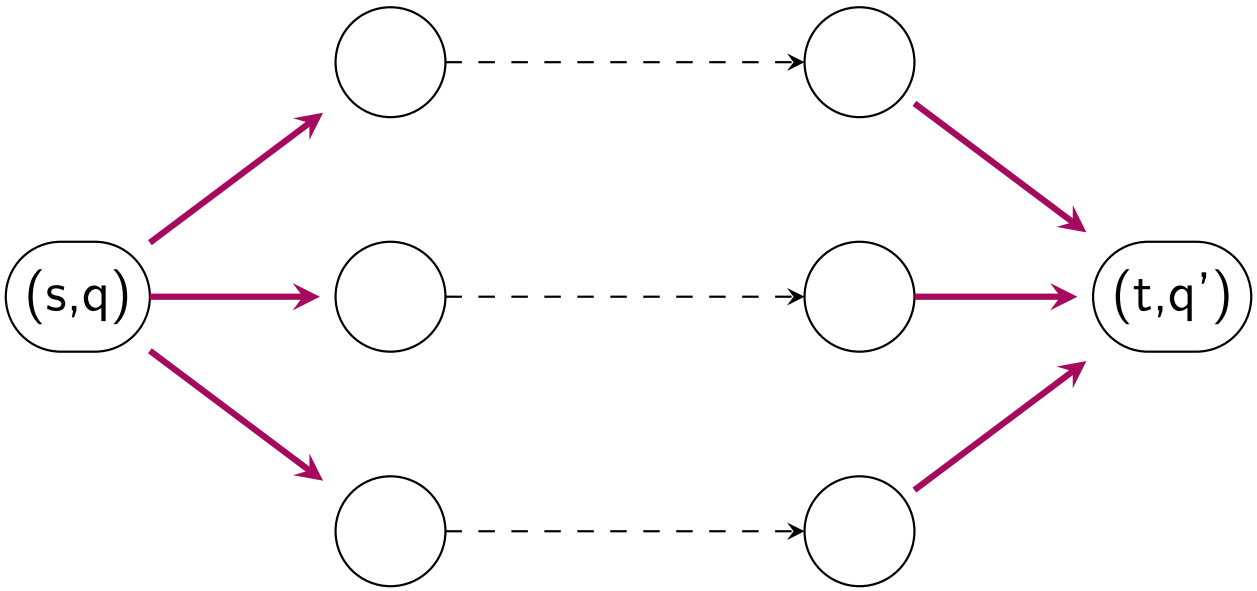

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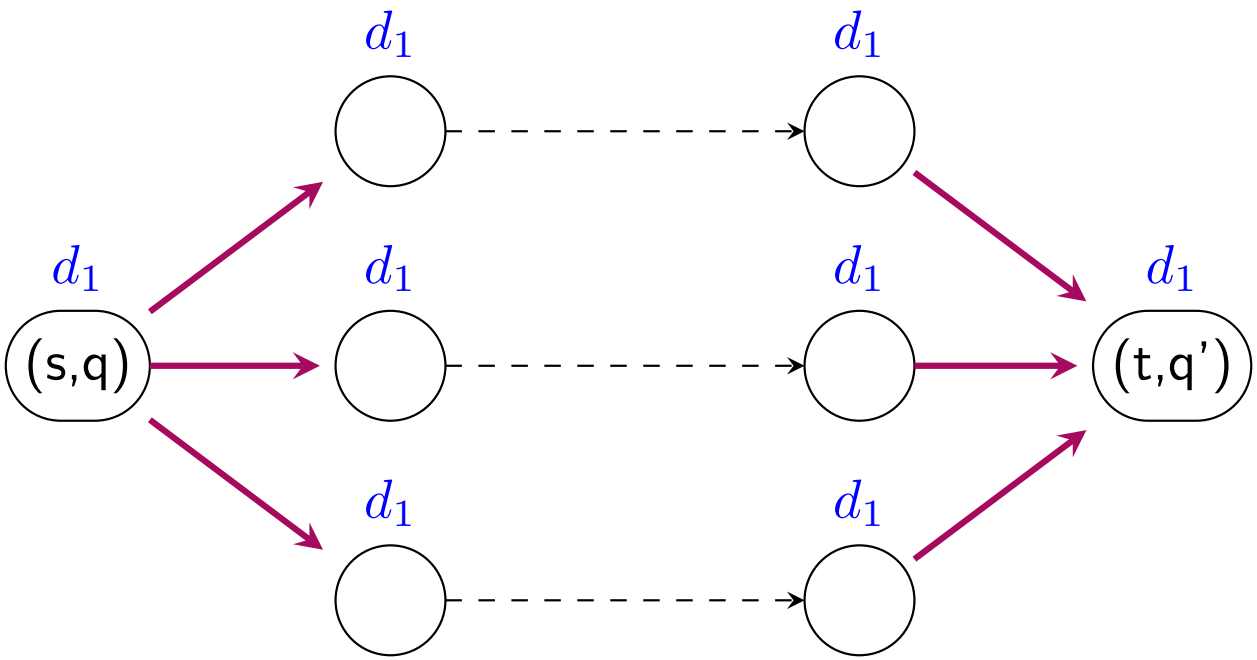

1. BFS on $D \times \mathcal{A}$ from sources to each vertex. Distances d_1

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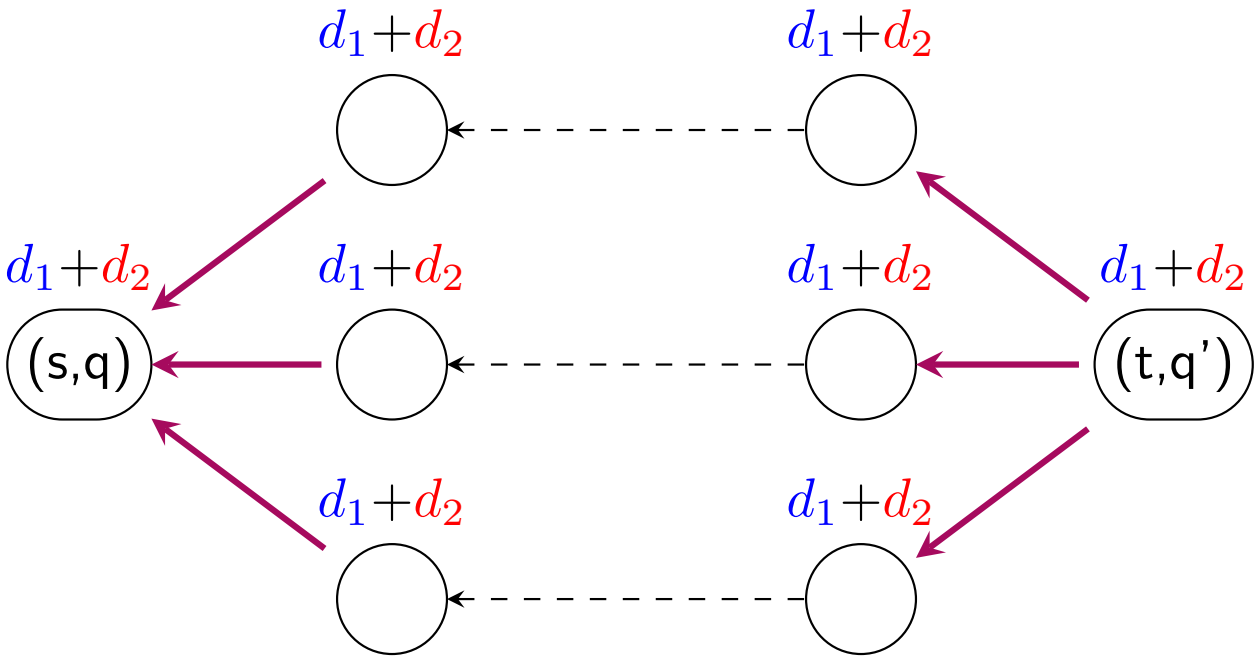

1. BFS on $D \times \mathcal{A}$ from sources to each vertex. Distances d_1
2. BFS on $(D \times \mathcal{A})^T$ from targets to each vertex. Mirror Distances d_2

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Proof.

3. $\exists v$ a vertex in w such that $\forall q \in Q, (d_1 + d_2)((v, q)) \geq |w|$? $w \in [\![\mathcal{A}]\!]_{SC}(D)$

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   If no such vertex exists: $w \notin [\![\mathcal{A}]\!]_{SC}(D)$

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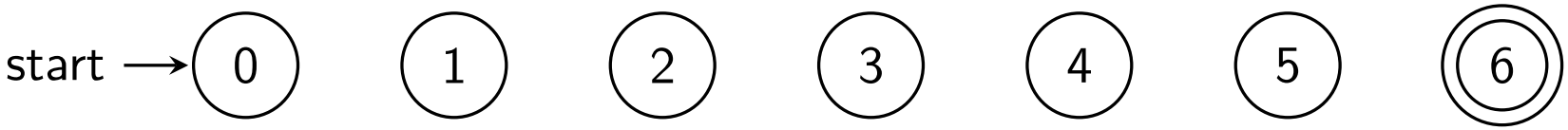

- $Q = \{i \mid 0 \leq i \leq |w| + 1\}$, $I = \{0\}$, $F = \{|w| + 1\}$.

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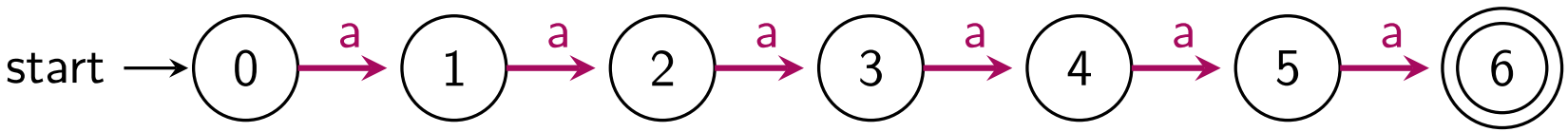

- $\delta = \{(i, a, i+1) \mid 0 \leq i \leq |w|, a \in \mathcal{A}\}$.

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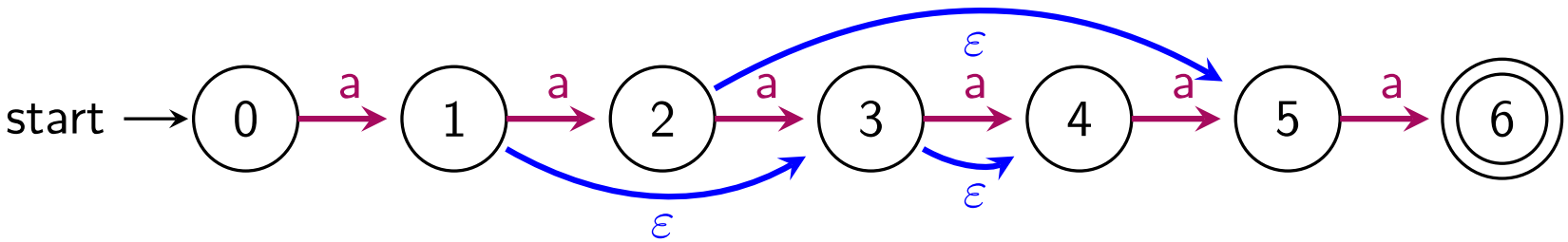

- $\delta = \{(i, a, i+1) \mid 0 \leq i \leq |w|+1, a \in \mathcal{A}\}$
  $\cup \quad \{(i, \varepsilon, j) \mid n_i = n_j, 0 \leq i < j \leq |w|+1, \nexists l \in]i, j[, n_l = n_i\}$.

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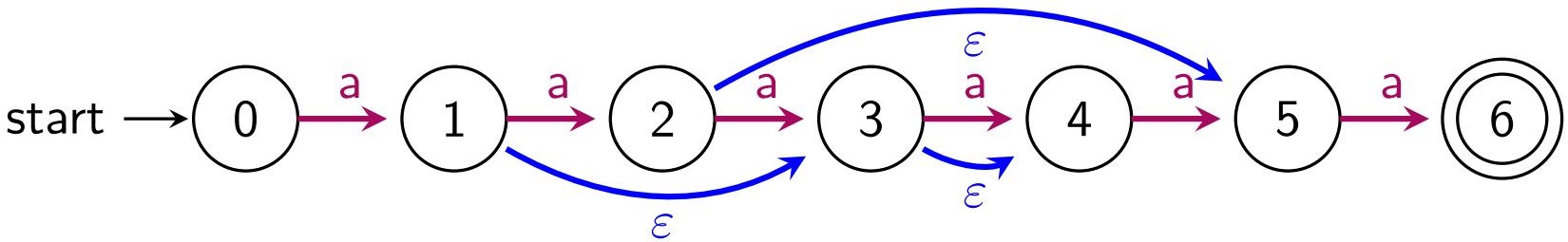

Lemma

$D_{\sqsubseteq w}$ has a size in $O(|w|)$ and can be constructed in $O(|w| \log |w|)$.

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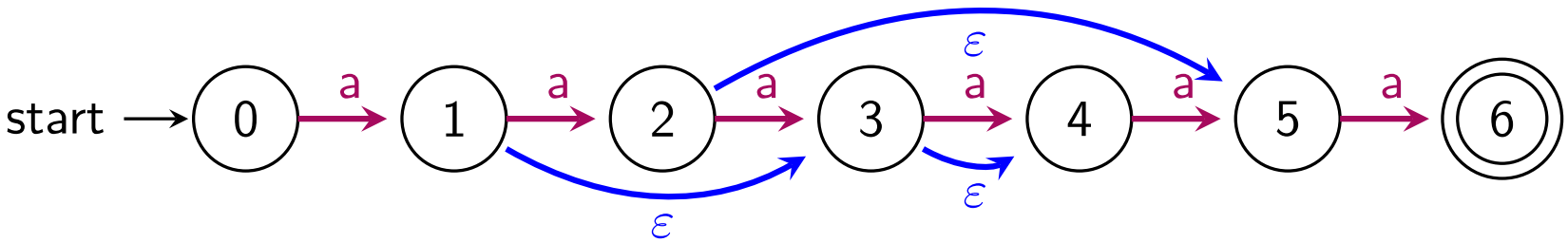

Lemma

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Lemma

$\mathcal{L}(D_{\sqsubseteq w}) = \{\text{LBL}(w') \mid w' \sqsubset w\}$.

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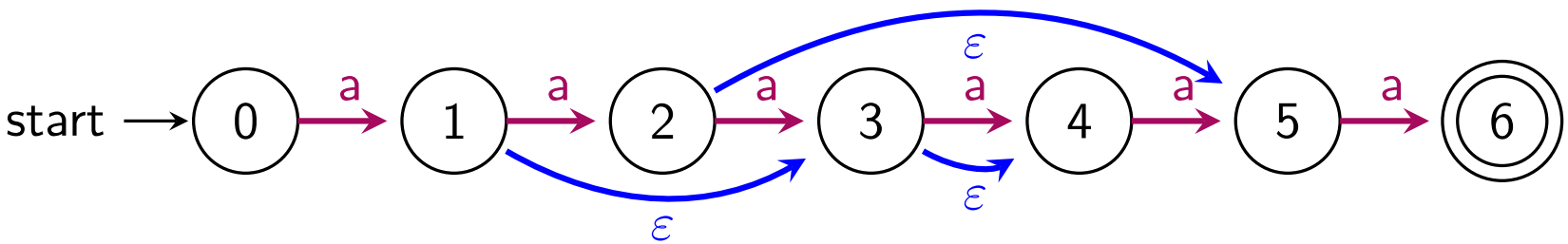

- Using a BFS on $D_{\sqsubseteq w} \times \mathcal{A}$, we check if $|w| = |shortestwalk|$?
  $w \in [\![\mathcal{A}]\!]_{MW}(D)$
  Otherwise, w is not a minimal walk. $w \notin [\![\mathcal{A}]\!]_{MW}(D)$

Computational Problems: Walk Enumeration

Problem definition

Walk Enumeration Under X Semantics

Input: A graph database D, a query $\mathcal{A}$, (s, t).
Output: $\{\!\{w \in [\![\mathcal{A}]\!]_X(D) \mid \textsc{EndPoints}(w) = (s, t)\}\!\}$.

Enumeration Complexity

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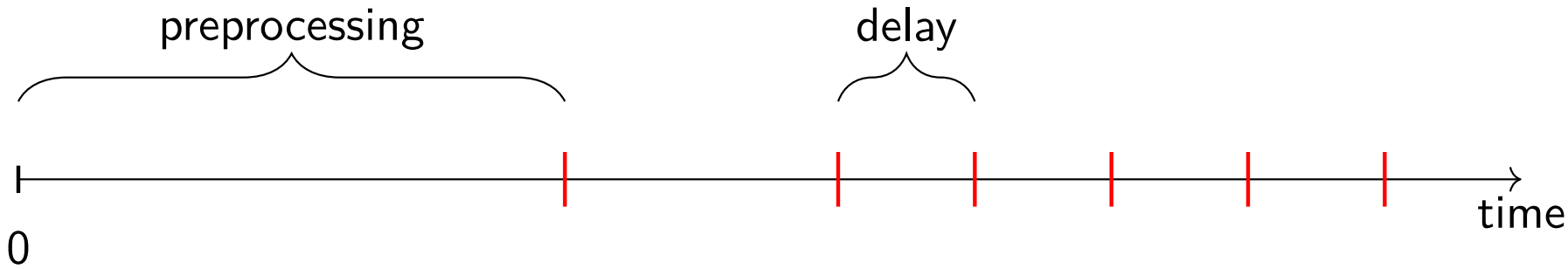

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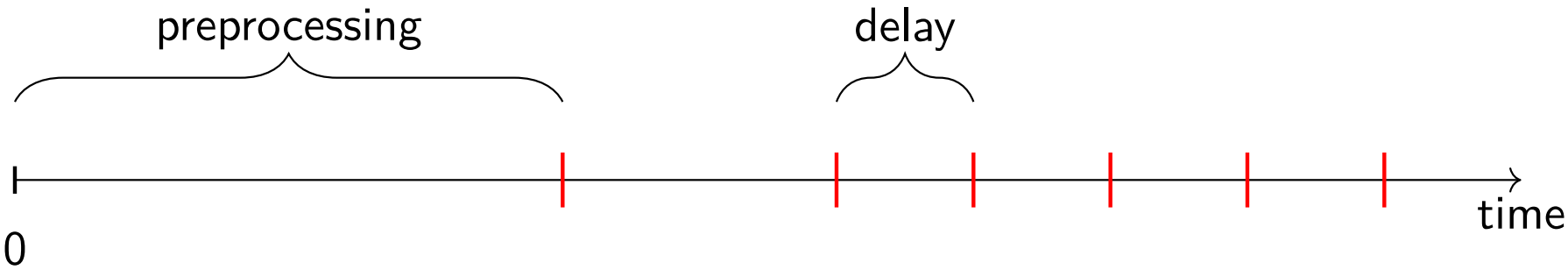

Definition

A problem A is in *DelayP* if there is a polynomial p and an algorithm which solves A on input x with a polynomial preprocessing and a delay in $O(p(|x|))$.

Walk Enumeration under shortest-coverage semantics

Theorem

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1. Use Floyd-Warshall to get the distances between all pairs of vertices.
2. For each vertex, find the states i, q, f that minimize the distance: $d((s,i),(v,q)) + d((v,q),(t,f))$.

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   - Output the walks.

Another Enumeration Problem

Distinct Walk Enumeration Under X Semantics

Input: A graph database D, a query $\mathcal{A}$, (s, t).
Output: $\{w \in [\![\mathcal{A}]\!]_X(D) \mid \textsc{EndPoints}(w) = (s, t)\}$.

Distinct Walk Enumeration

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A general approach: DFS on the graph database.

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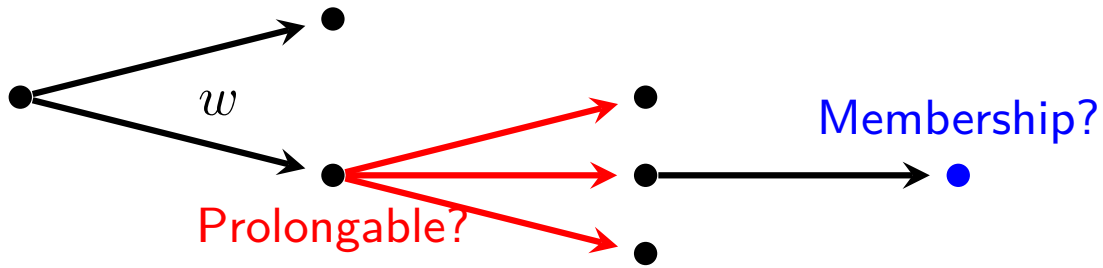

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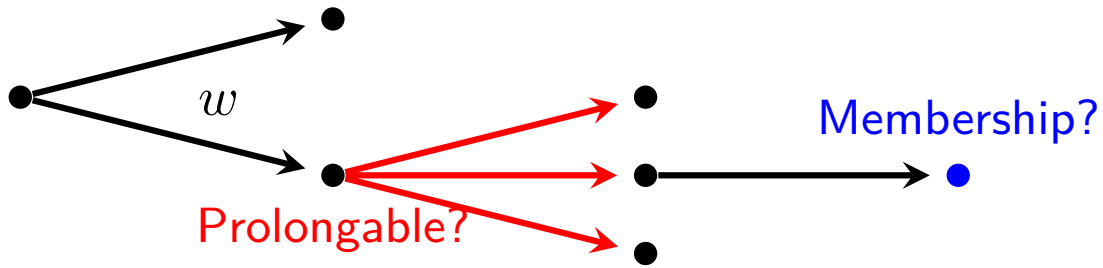

Theorem

If Prolongability $\in$ ***P*** *and* Walk Membership $\in$ ***P***, *then* Distinct Walk Enumeration $\in$ ***DelayP***.

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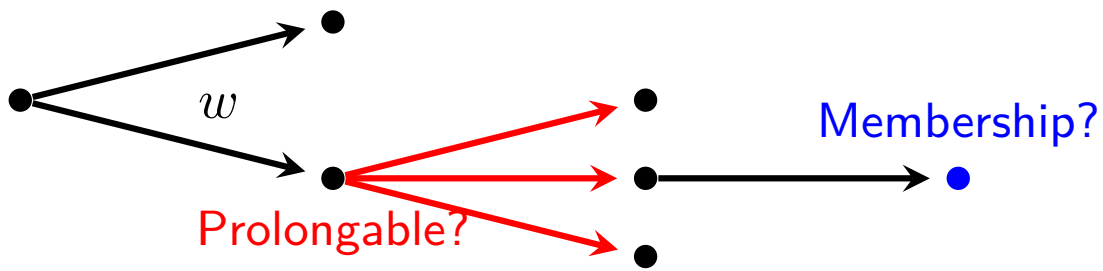

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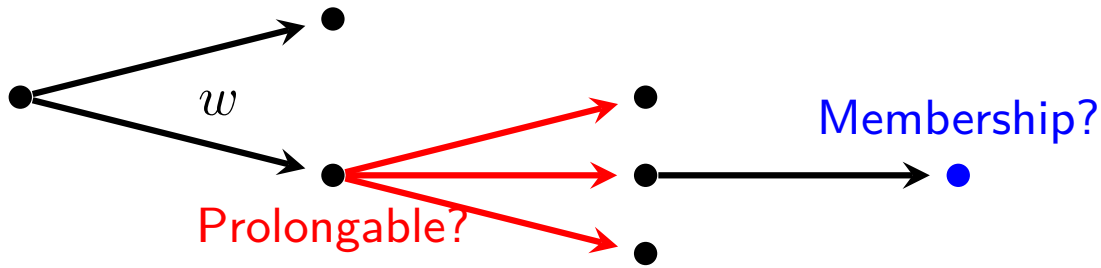

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(Distinct) Walk Enum. under minimal-walk semantics

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Open problem:

Is (Distinct) Walk Enum. under minimal-walk semantics in **DelayP**?

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Prolongability *under minimal-walk semantics is NP-complete.*

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Proof. Reduction from Two-Disjoint-Paths.

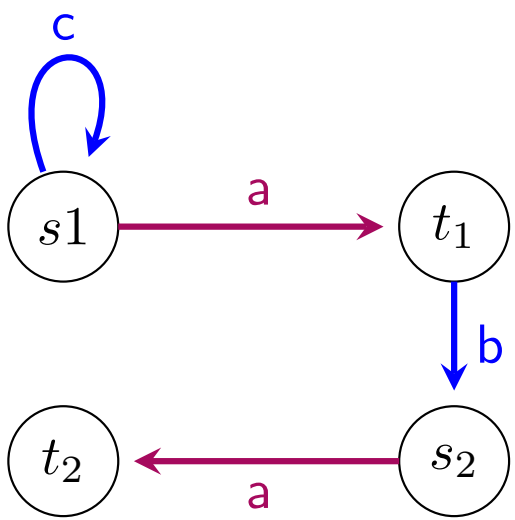
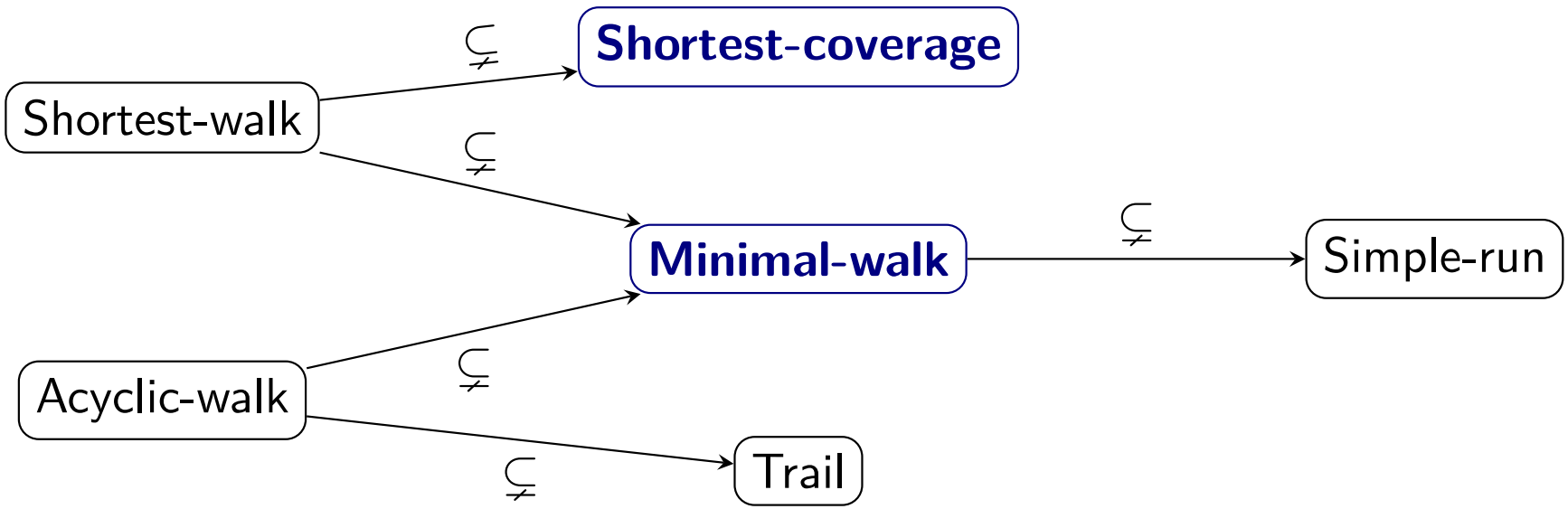


Figure: Reduction with the query $(c \cdot a^* \cdot b \cdot a^*) + a^*$

Conclusion

Summary:

	Shortest-coverage	**Minimal-walk**
Coverage	Vertex coverage	Subwalk coverage
Tuple Membership	NL-complete	NL-complete
Walk Membership	P	P
Walk Enumeration	DelayP	Open
Distinct Walk Enumeration	DelayP	Open



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- Applications to query languages: Cypher, GQL.

Thank you!